

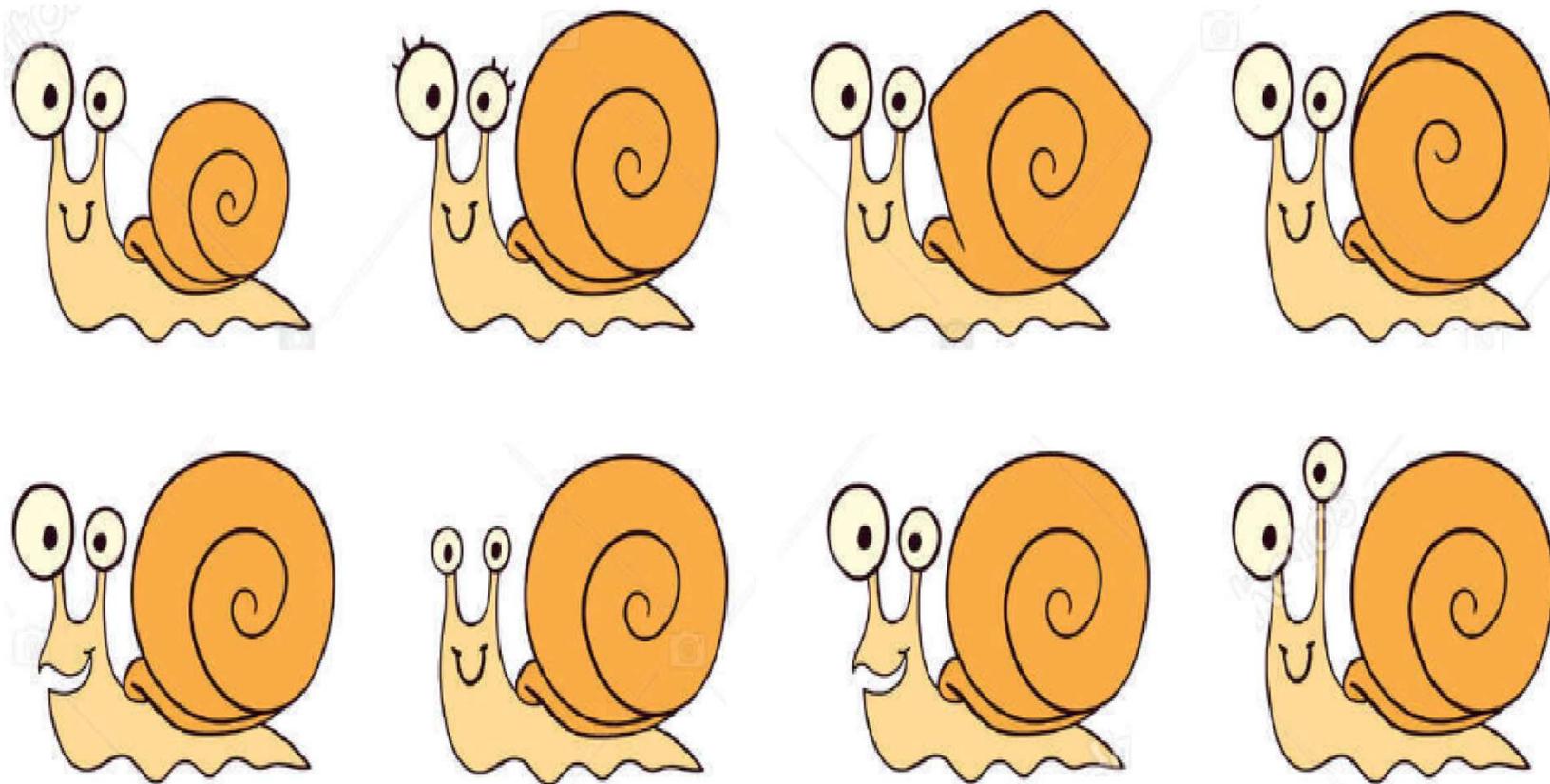
Topological invariants applied to the classification of phase portraits of quadratic differential systems.

Joan C. Artés & Maria Morales (TFG)

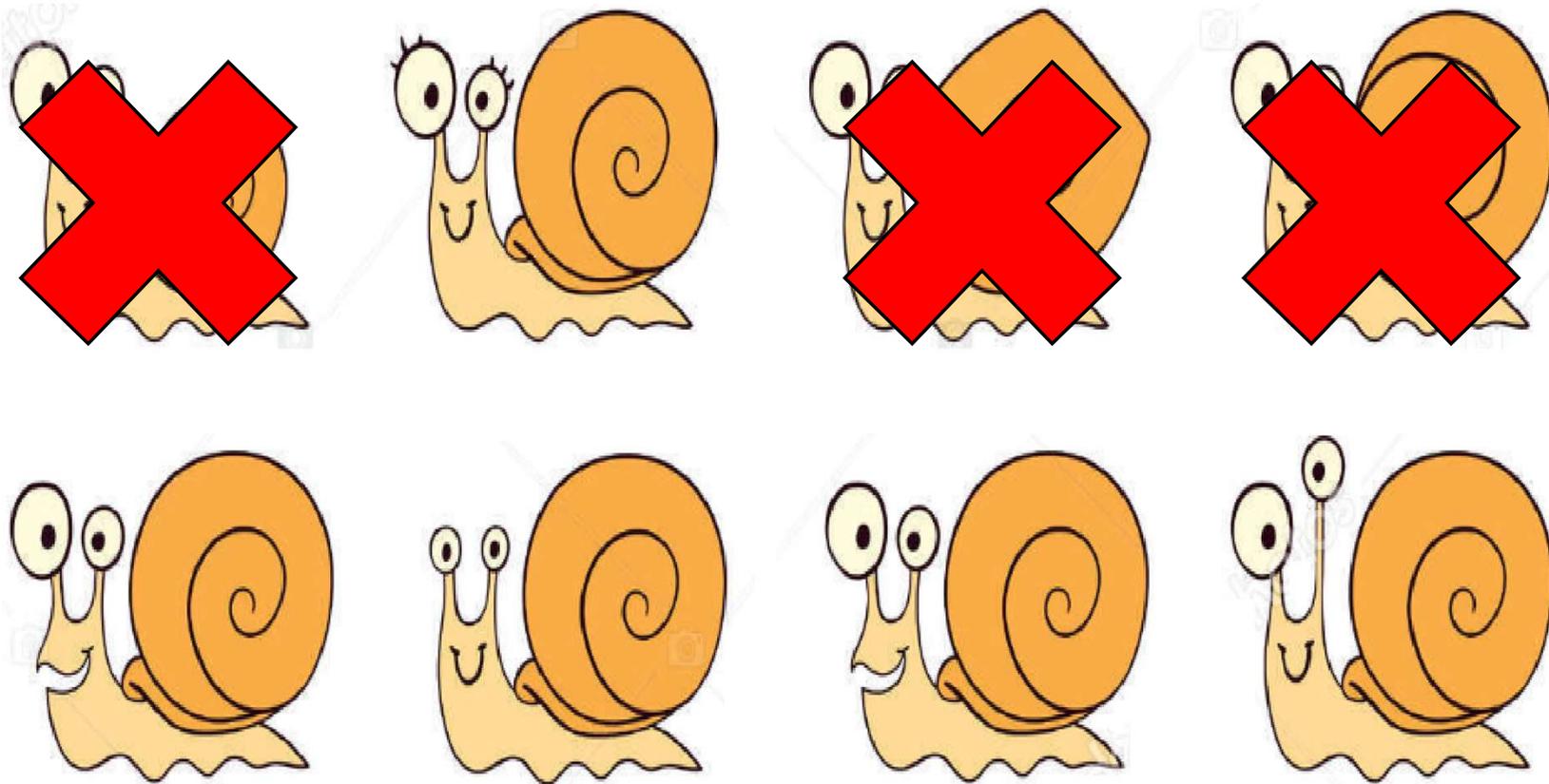
Universitat Autònoma de Barcelona

Colaborators: Belfar, Benabdallah, ... <50>..., Zegeling, Zhang Xiang

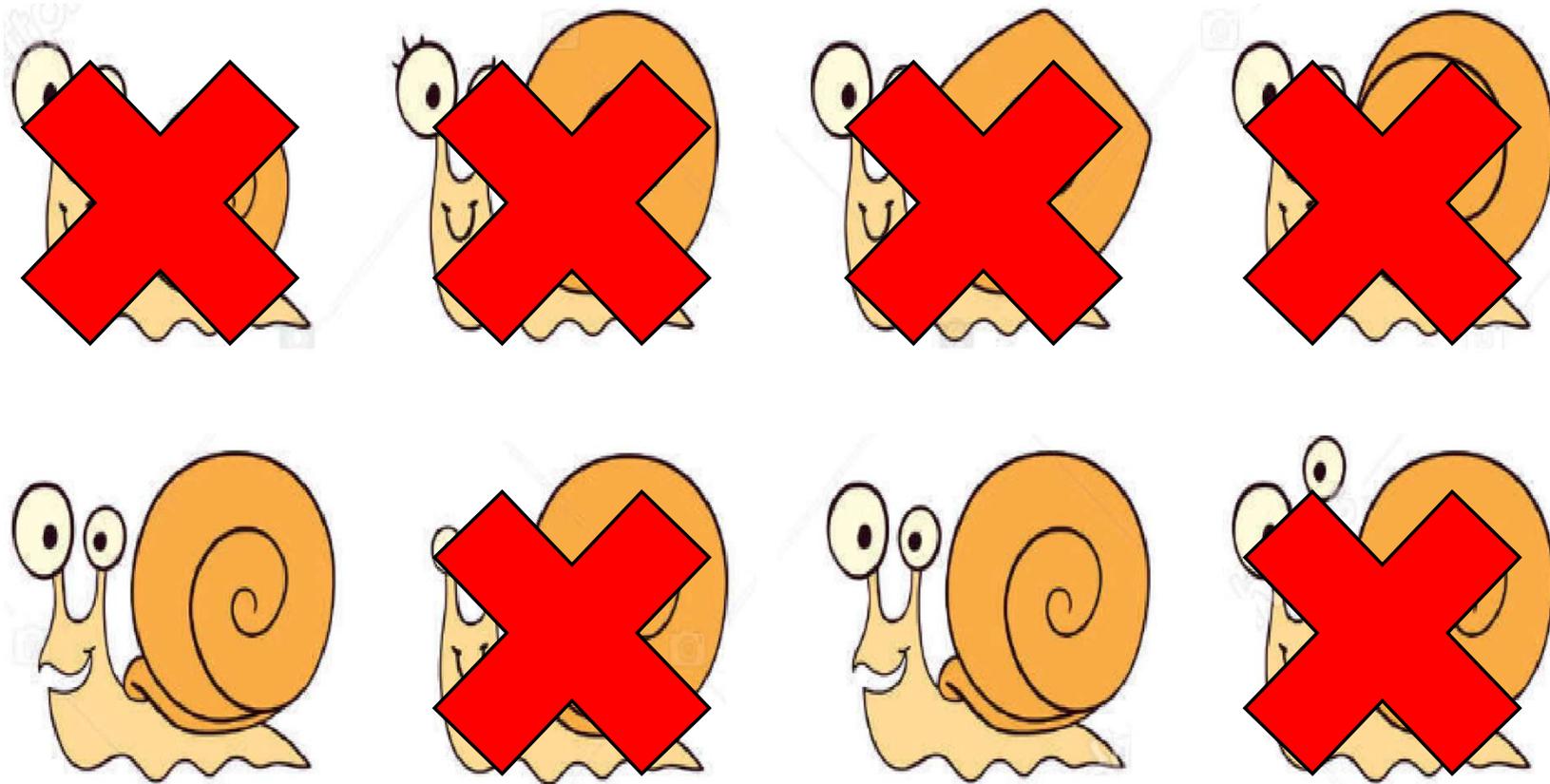
A “DIFFICULT” GAME



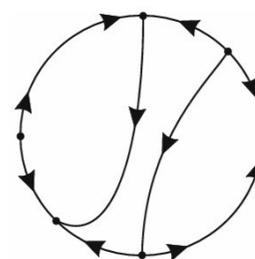
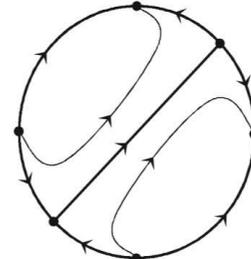
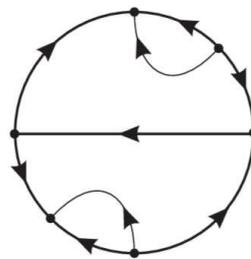
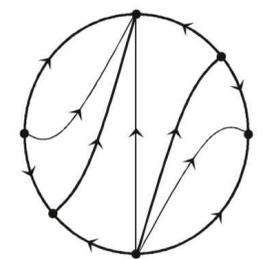
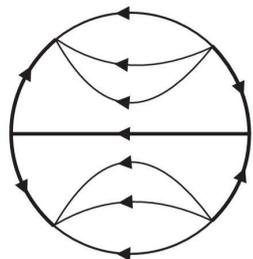
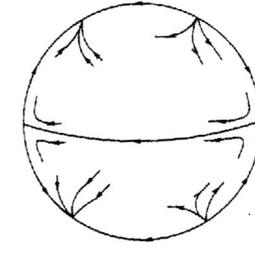
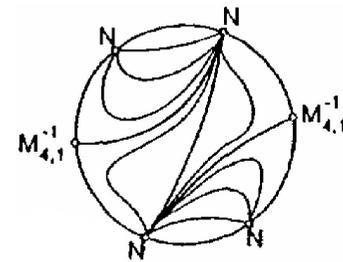
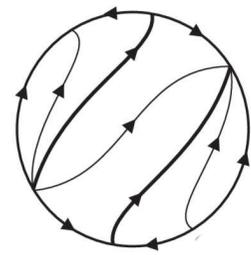
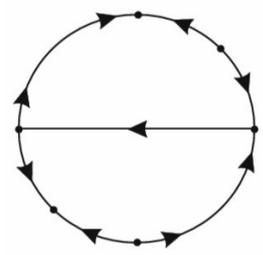
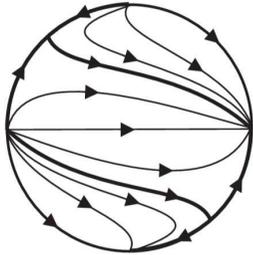
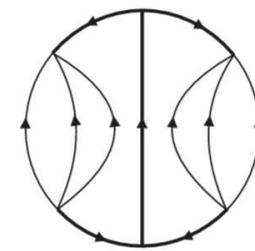
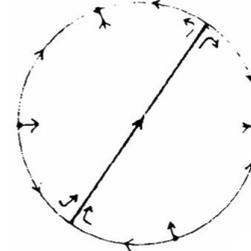
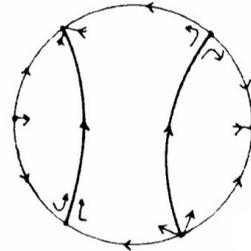
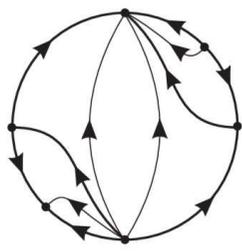
A “DIFFICULT” GAME (2)



A “DIFFICULT” GAME (3)



NOW, THIS IS OUR SERIOUS PROBLEM

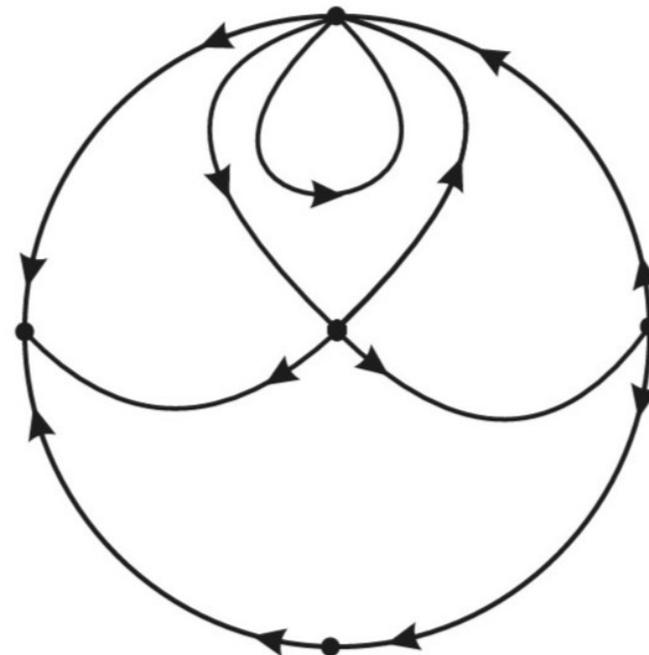


NOTATION FINITE SINGULARITIES

- a : antisaddle
 - f : focus
 - n : node
- s : saddle
- c : center
- sn : saddle-node
- cp : cusp
- es : elliptic saddle
- $phph$: two parabolic sectors and two hyperbolic ones
- hh : two hyperbolic sectors
- $hhhhh$: six hyperbolic sectors
- ee : two elliptic sectors

NOTATION INFINITE SINGULARITIES

- N : node
- S : saddle
- $\begin{pmatrix} f \\ i \end{pmatrix} SN$: saddle-node
- Sectors
 - E : elliptic
 - H : hyperbolic
 - P : parabolic



$$sn; \begin{pmatrix} 1 \\ 2 \end{pmatrix} E - H, \begin{pmatrix} 1 \\ 1 \end{pmatrix} SN$$

NOTATION INFINITE SINGULARITIES (2)

- $PHP - E$
- $HHH - H$
- $HHP - P$
- $HE - P$
- $H - H$
- $HE - PHH$
- $HH - P$
- $EH - HE$
- $E - H$
- $E - E$
- $PH - PH$
- $PH - H$
- $E - PH$
- $EE - P$
- $PH - HP$
- $EE - HH$
- $PHP - PHP$
- $HHH - HHH$
- $HHP - PHH$
- $P - P$
- $HHH - HP$
- $HPH - P$
- $E - HHH$
- $EH - P$

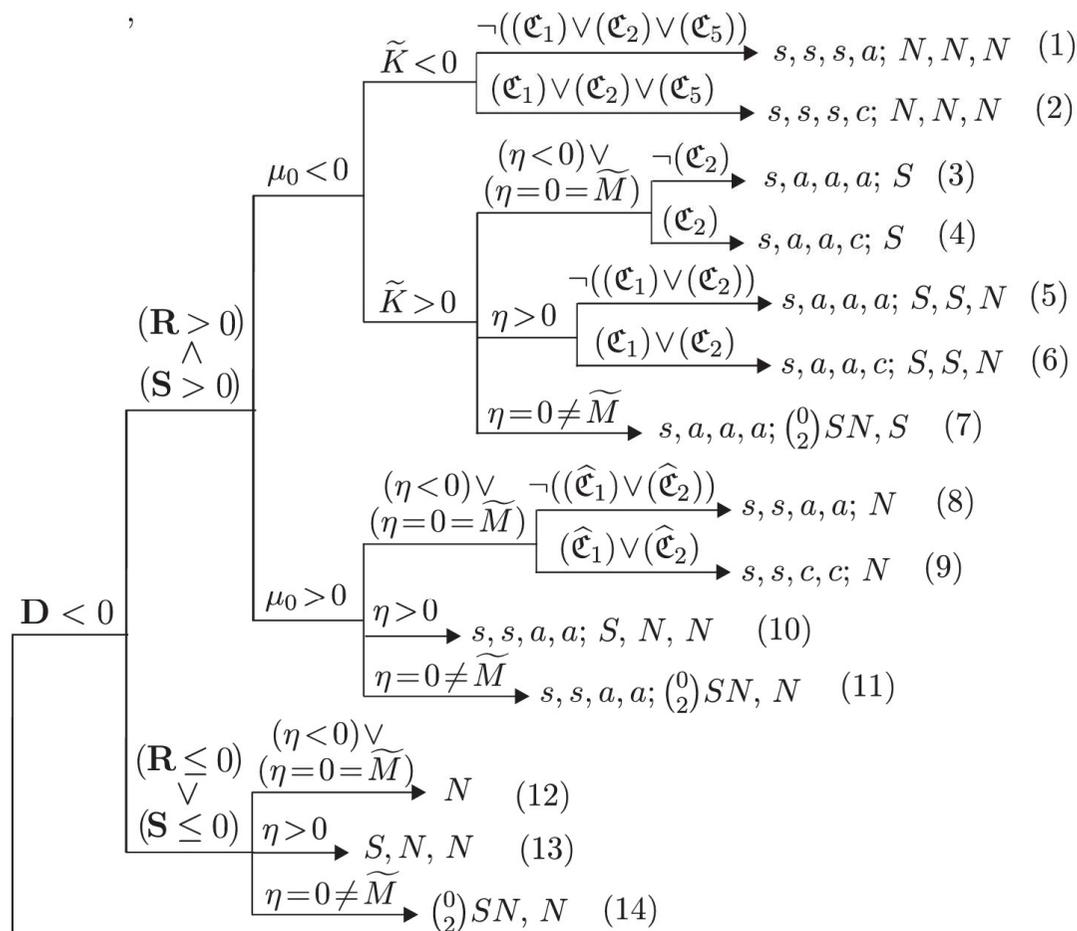
NOTATION DEGENERATE CASES

- $[\infty; \emptyset]$
- $[\infty; N]$
- $[\infty; N^d]$
- $[\infty; S]$
- $[\infty; C]$
- $[\infty; \binom{2}{0}SN]$
- $[\infty; \binom{3}{0}ES]$
- $\Theta [||]$: real straight line lines intersecting
- $\Theta [^\circ]$: real ellipse
- $\Theta [^\odot]$: complex ellipse
- $\Theta [>()]$: hyperbola
- $\Theta [U]$: parabola
- $\Theta [X]$: two intersecting straight lines
- $\Theta [.]$: two complex straight lines intersecting
- $\Theta [|||]$: two real paralel straight lines
- $\Theta [|||^c]$: two complex parallel straight lines
- $\Theta [||_2]$: double real straight line

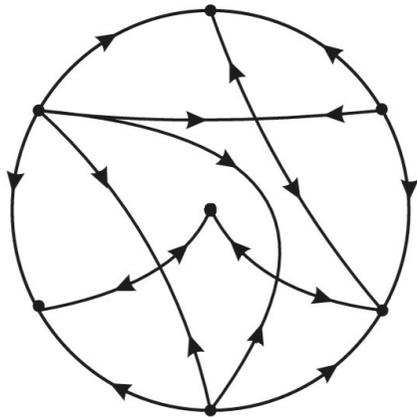
TOPOLOGICAL CONFIGURATIONS

4

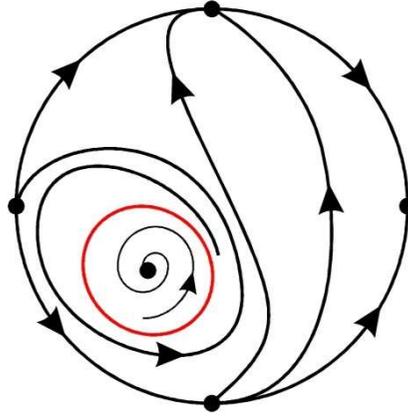
J.C. ARTÉS, J. LLIBRE, D. SCHLOMIUK AND N. VULPE



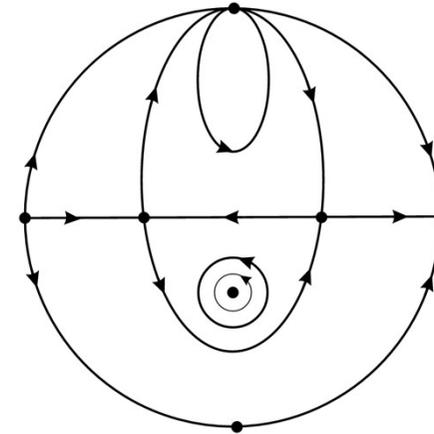
TOPOLOGICAL INVARIANTS



a) $s, s, s, a; N, N, N$



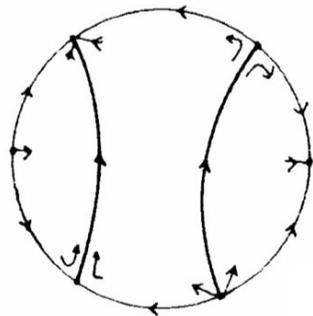
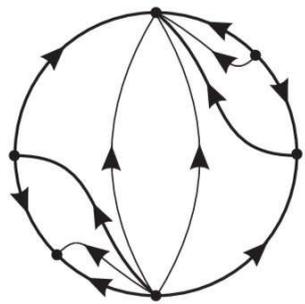
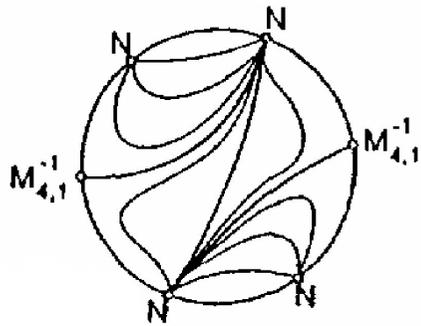
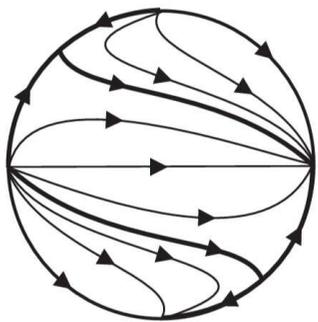
b) $a; \begin{pmatrix} 1 \\ 1 \end{pmatrix} SN, \begin{pmatrix} 0 \\ 2 \end{pmatrix} SN$



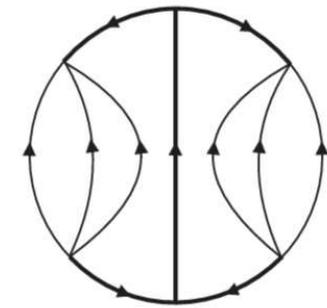
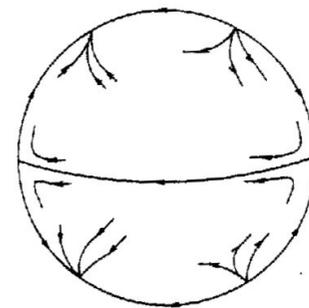
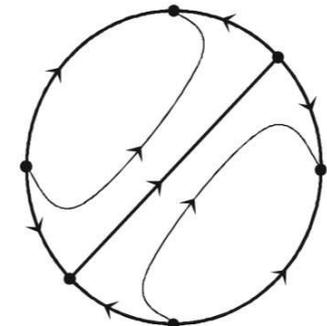
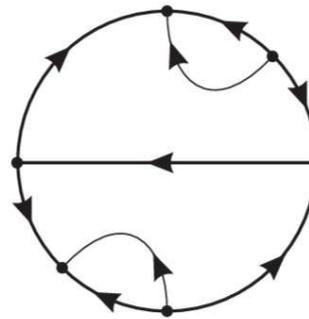
c) $s, s, c; \begin{pmatrix} 1 \\ 2 \end{pmatrix} E - H, N$

	TCSP	FIN SEP	INF SEP	LC	
a)	1	4442	312211		
b)	86	1	2120	1	
c)	78	440	2101		

THE ANSWER TO THE PREVIOUS PROBLEM

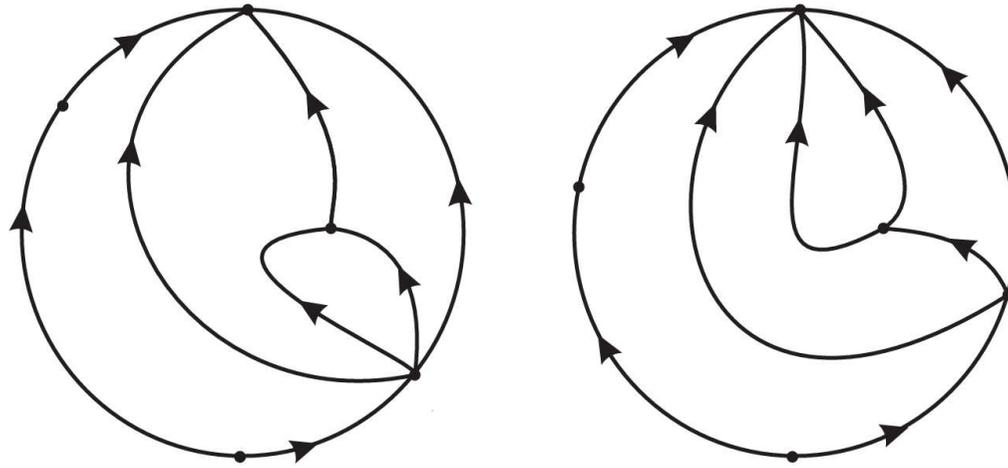


13) $\emptyset; S, N, N$ 110110



13) $\emptyset; S, N, N$ 100100

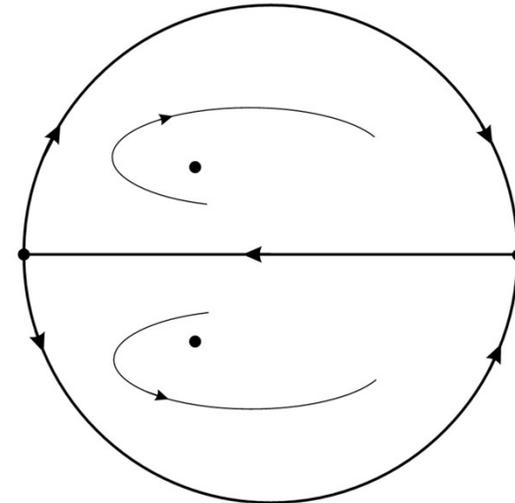
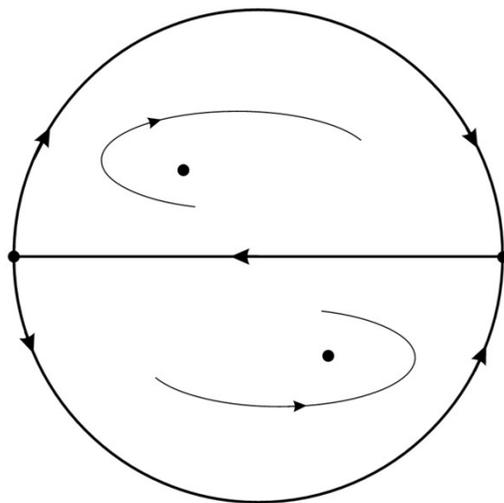
TOPOLOGICAL INVARIANTS (2)



a) $sn; \binom{2}{2} PH - H, N$

	TCSP	FIN SEP	INF SEP	LC	BASIN	
a)	152	3	3200		To SN	
b)	152	3	3200		To N	

TOPOLOGICAL INVARIANTS (3)



$a, a; S$

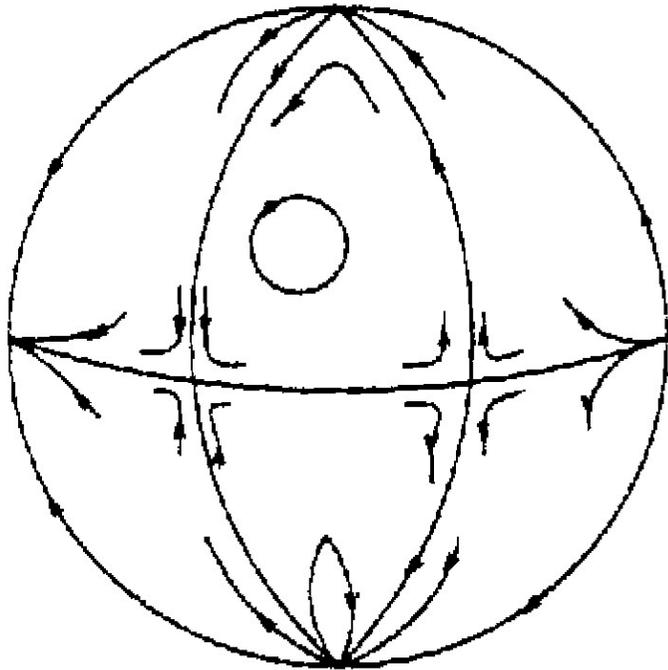
	TCSP	FIN SEP	INF SEP	LC	DUAL	
a)	16	00	11		Dif	
b)	16	00	11		Same	

DATA COLLECTED UP TO NOW

- 80 articles (since 1983 to nowadays).
- 5759 phase portraits.
- 1174 topologically different modulo limit cycles.
- 173 with limit cycle(s) in the article (but there are many more).
- All 207 configurations of singularities with some representative.
- 182 configurations completely studied, 25 on work.
- 114 configurations with a unique phase portrait.
- One configuration: $39) s, a, sn; S, (0,2)SN$ with 99 phase portraits.

HELPS TO DETECT ERRORS

The next phase portrait does not exist



- In the same way, there have been found some minor typos and more serious mistakes in some of the papers collected.

MISSED PORTRAITS

- Papers that claim that have completed some subclass of phase portraits can be compared with others to check if they have missed some.
- New papers close to be finished may be compared with the database to see if they have obtained some new phase portrait, and in case it seems so, recheck to confirm its existence (so to avoid producing a false phase portrait).
- The database also contains a list of potential phase portraits which have already been proved to be impossible.
- There remain some potential phase portraits for which there is not yet an example, neither a proof of their impossibility.

CODIMENSION

- For a better classification, the topological codimension modulo limit cycles of singularities, configurations of singularities and phase portraits is used.
- The definition is mainly based on Sotomayor's definition:
- Let B_0 be the set of polynomial vector fields of degree n . Let A_0 be the subset of B_0 formed by all the fields which are structurally stable. We will assign codimension 0 to the fields of A_0 . Let $B_1 = B_0 \setminus A_0$. This will be called the "bifurcation set". Let A_1 be the subset of B_1 formed by all the fields which are structurally stable inside B_1 . We will assign codimension 1 to the fields of A_1 . And the definition continues recursively.
- A small improvement to it was needed to cover some special cases.

CODIMENSION (2)

- Thus we have configurations of singularities of codimension 0 like:

1) $s, s, s, a; N, N, N$ 8) $s, s, a, a; S, N, N$ or 16) $a, a; N$

- Configurations of codimension 1 like:

22) $a, a; S, \binom{0}{2}SN$ 37) $s, a, sn; N$ or 73) $s, a, a; \binom{1}{1}SN$

- Configurations of codimension 2 like:

32) $a, a, sn; S, \binom{0}{2}SN$ 48) $cp; N$ 51) $sn, sn; S, N, N$ 86) $a; \binom{1}{1}SN, \binom{0}{2}SN$ or 91) $a; \binom{1}{2}PHP - E, S$

- Configurations of codimension 4 like:

55) $cp, sn; N, \binom{0}{2}SN$ 69) $hhhhhh; N, N, N$ 83) $a, s, s; [\infty, \emptyset]$ or 111) $a, sn; \binom{1}{3}HHP - H$

- Up to one configuration of codimension 9.

CODIMENSION (3)

- The codimension of a configuration of singularities is not always the plain sum of codimensions of its singularities. At some high level of codimension, interdependence appears.
- To the codimension provided by the configuration of singularities, we must add the codimension produced by separatrix connections. Also interdependence may appear.
- Theorem: The maximum number of affine separatrix connections in quadratic systems is 2 (with the exception of two cases with centers). (To be published).

STATE OF THE RESEARCH

GLOBAL CODIMENSION PHASE PORTRAIT

	0	1	2	3	4	5	6	7	8	9
CODIMENSION	0	44	61	≥ 9	0	0	0	0	0	0
OF THE	1	X	141	≥ 175	≥ 3	0	0	0	0	0
CONFIGURATION	2	X	X	≥ 189	≥ 160	≥ 16	0	0	0	0
OF	3	X	X	X	124	69	5	0	0	0
SINGULARITIES	4	X	X	X	X	54	14	0	0	0
	5	X	X	X	X	X	33	1	0	0
	6	X	X	X	X	X	X	18	0	0
	7	X	X	X	X	X	X	X	8	0
	8	X	X	X	X	X	X	X	X	3
	9	X	X	X	X	X	X	X	X	X

PLUS 31 WITH CENTERS

VISIT: DATABASE AND ENCYCLOPEDIA

- Let us make a visit to the data base which is now in an Excel file
- And we see also a small sketch of the Encyclopedia.

THANKS FOR YOUR
ATTENTION