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## ON THE INTEGRABILITY OF THE PROJECTIVE EQUATIONS

In this talk we consider integrability of a family of second-order differential equations that is a projection of a geodesic flow of a two-dimensional (pseudo) Riemannian manifold. We develop two approaches for establishing integrability of the projective equations. The first one is based on finding solution of equivalence problems for the family of projective equations and its integrable subcases. The second one is connected with the classification of quasi-polynomial invariants for the projective equations. We introduce the notion of generalized Darboux first integrals for Hamiltonian systems for geodesics and demonstrate how these integrals are connected to the quasi-polynomial invariants of the projective equations. We construct several families of integrable Riemannian metrics with generalized Darboux first integrals, which at the particular values of the parameters degenerate into rational first integrals of arbitrary degree. The talk is based on three recent works with Jaume Giné [1, 2, 3].

## Referencias

- [1] J. Giné, D.I. Sinelshchikov, On the geometric and analytical properties of the anharmonic oscillator, Commun. Nonlinear Sci. Numer. Simul. 131 (2024) 107875. doi: 10.1016/j.cnsns.2024.107875
- [2] J. Giné, D. Sinelshchikov, Integrability of Oscillators and Transcendental Invariant Curves, Qual. Theory Dyn. Syst. 24 (2025) 26. <https://doi.org/10.1007/s12346-024-01182-x>.
- [3] J. Giné, D. Sinelshchikov, Metrisable oscillators and (super)integrable two-dimensional metrics, submitted to J. Geom. Phys.