

On convergence of normal form transformations

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We discuss some aspects concerning transformations of local analytic, or formal, vector fields to Poincare-Dulac normal form, and the convergence of such transformations. We first mention A.D. Bruno's approach to formal normalization [1], as well as convergence results in presence of certain (simplified) versions of Bruno's "Condition A", and along the way we also identify a large class of systems that satisfy Bruno's diophantine "Condition omega". We retrace the proof steps in Bruno's work, using a different formalism. We then proceed to show how Bruno's approach naturally extends to an elementary proof of L. Stolovitch's formal and analytic simultaneous normalization theorems for abelian Lie algebras of vector fields [2].

The talk is based on the recent work with Sebastian Walcher [3].

References

1. A. D. Brjuno (Bruno), The analytical form of differential equations. Trans. Mosc. Math. Soc., 25:131-248 (1972).
2. L. Stolovitch, Singular complete integrability. IHES Publ. Math. 91:133-210 (2000).
3. V. G. Romanovski, S. Walcher, On convergence of normal form transformations, <https://arxiv.org/abs/2510.00925>.