Henryk Żołądek   University of Warsaw, Poland

A qualitative approach to hydrodynamics

I propose an apparently novel approach to hydrodynamic type evolution equations.

I demonstrate (on an example) that the standard procedure of elimination of the pressure from the Navier–Stokes equations is qualitatively incorrect. Namely, the two conditions for the fluid velocity, zero divergence and vanishing at the boundary, are not compatible from the physical point of view.

I propose other scenarios of elimination of the pressure in dimensions two and three. They rely upon splittings of the space of vector fields into two summands, one of which consists of divergence free vector fields. The solutions to the initial value problem for the Navier–Stokes system exist (under some smallness assumption), but are not unique. This provides a substantial contribution to one of the millennium problems.

In the cases of Burgers equation and the reaction diffusion equation I show that, if the initial condition is sufficiently small (in a precise sense), then the solution exists for all positive times and is unique. On the other hand, there exist situations when the solutions either cease to be unique or blow up to infinity.