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O-minimal Theory: A tool to study the finiteness of limit cycles of piecewise vector fields

The second part of Hilbert's sixteenth problem has inspired many significant advances in the qualitative theory of differential equations [1]. Motivated by this classical problem, we study its analogue in the setting of piecewise vector fields. Given a collection of r horizontal and s vertical lines, consider the partition $\mathcal{P}_{(r,s)}$ of the plane into $k(r,s) = 4rs + 2(r+s) + 1$ disjoint cells,

$$\mathbb{R}^2 = \bigsqcup_{i=1}^{k(r,s)} C_i,$$

where each cell is of dimension 0(points), 1(segments) or 2(rectangles). Let $PC^2(\mathcal{P}_{(r,s)})$ denote the space of piecewise constant vector fields associated with this partition.

We will explain the proof of the following theorem:

Theorem 1 *: There exists a natural number $N(r,s)$ such that each vector field in $PC^2(\mathcal{P}_{(r,s)})$ has at most $N(r,s)$ semi-limit cycles.*

This theorem is a first step towards the conjecture stated in [2] for the more general class $PL^2(\mathcal{P}_{(r,s)})$ of piecewise linear vector fields over a partition $\mathcal{P}_{(r,s)}$, and our approach extends the result of [3]. For this purpose, the main tool we used was the o-minimal theory [4], which allowed us to obtain the finiteness of the limit cycles in this setting.

Referencias

[1] Y. Ilyashenko, "Centennial history of hilberts 16th problem," *Bulletin of the American Mathematical Society*, vol. 39, no. 3, pp. 301–354, 2002.

[2] D. Panazzolo, "Tame semiflows for piecewise linear vector fields," *Annales de l'institut Fourier*, vol. 52, no. 6, pp. 1593–1628, 2002.

[3] V. Carmona, F. Fernandez-Sanchez, and D. D. Novaes, "Uniform upper bound for the number of limit cycles of planar piecewise linear differential systems with two zones separated by a straight line," *Applied Mathematics Letters*, vol. 137, p. 108501, 2023.

[4] L. Van den Dries, *Tame topology and o-minimal structures*, vol. 248. Cambridge university press, 1998.