V Symposium on Planar Vector Fields

Lleida, January 12-16, 2026

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00-10:00	Registration	Zoladek	Christopher	Hofbauer	Zhang
10:00-11:00	Dumortier	Carmona	Buica	Panazzolo	De Maesschalck
11:00-11:30	Coffee-Break	Coffee-Break	Coffee-Break	Coffee-Break	Coffee-Break
11:30-12:15	Marín	García	Novaes	von Bothmer	Artés
12:15-13:00	Romanovsky	Sinelshchikov	Ponce (*)	Mello	Gasull (*)
13:00-15:00	Lunch	Lunch	Lunch	Lunch	Lunch
15:00-15:45	Carvalho	Teruel		Llibre (**)	
15:45-16:30	Rojas	Posters		In memorian act (***)	
16:30-17:15	Rodero				

The sessions will be mainly in Room 1.04 of Escola Politècnica Superior (EPS) building. Except the posters session (Room 0.29 of Edifici Polivalent 1) and the sessions on Thursday afternoon (Room 2.03 – Sala de Graus of EPS).

(*) The invited talks of Wednesday and Friday (after Coffee-Break) end at 13:15.
(**) The invited talk of Thursday afternoon ends at 16:00.
(***) The in memorian act starts at 16:15.

Book of Abstracts

Freddy Dumortier Hasselt University, Belgium

More on multi-layer canard cycles

Abstract. Given a slow-fast system containing several layers, we study the canard cycles transiting through these layers. All the canard cycles we consider are topological circles and in between the layers we restrict to generic Hopf breaking mechanisms and generic jump breaking mechanisms. As shown in previous talks, there is a difference in between transiting a terminal layer or transiting a dodging layer, but maybe more important is the precise way in which successive layers are connected to each other at the breaking mechanism. In order to study the precise passage at such a breaking mechanism we have to compare a transition map coming from an attracting sequence on one side to the transition map coming (in negative time) from a repelling sequence on the other side. The comparison can be side-preserving or side-reversing and this makes a major difference in the calculations, affecting strongly the results on the number of limit cycles and their possible bifurcations. As already explained in previous talks the equations to consider merely depend on the so-called “connection diagram” associated to the multi-layer canard cycle. We will provide some general results concerning these equations, leading to precise consequences on the cyclicity and the bifurcations, with emphasis on 2-layer canard cycles. We will present the essential ingredients of the proofs. We prove that all possible abstract connection diagrams can be realized by a multi-layer canard cycle of a planar polynomial vector field. We also precisely state the conditions under which a connection diagram can be realized by a multi-layer canard cycle of a classical Liénard equation as well as of a generalized polynomial Liénard equation. We end the talk by giving an overview of problems that we working at.

Based on joint work with Peter De Maesschalck and Robert Roussarie.

David Marín Universitat Autònoma de Barcelona, Spain

Derivatives of the Separation Function of Generalized Saddle Connections

Abstract. A classical formula shows that the breaking of a connection between two hyperbolic saddles can be studied by means of a convergent integral that is often called the Melnikov integral. In this talk we will explain how to apply this formula to more general situations, for instance, when the singularities are semi-hyperbolic or even nilpotent. We will see that in some of these cases, the improper integral is no longer convergent but nevertheless, under convenient hypothesis, there is a kind of residue that provides the desired information. We will present a general statement and some examples illustrating its applicability.

This is a joint work with Jordi Villad+elprat.

Valery G. Romanovski University of Maribor and CAMTP, Slovenia

On convergence of normal form transformations

Abstract. We discuss some aspects concerning transformations of local analytic, or formal, vector fields to Poincare-Dulac normal form, and the convergence of such transformations. We first mention A.D. Bruno's approach to formal normalization [1], as well as convergence results in presence of certain (simplified) versions of Bruno's "Condition A", and along the way we also identify a large class of systems that satisfy Bruno's diophantine "Condition omega". We retrace the proof steps in Bruno's work, using a different formalism. We then proceed to show how Bruno's approach naturally extends to an elementary proof of L. Stolovitch's formal and analytic simultaneous normalization theorems for abelian Lie algebras of vector fields [2].

The talk is based on the recent work with Sebastian Walcher [3].

References

[1] A. D. Brjuno, "The analytical form of differential equations." *Trans. Mosc. Math. Soc.*, 25 (1972) 131–248.

[2] L. Stolovitch, "Singular complete integrability." *IHES Publ. Math.* 91 (2000) 133–210.

[3] V. G. Romanovski and S. Walcher, "On convergence of normal form transformations." https://arxiv.org/abs/2510.00925.

Yagor R. Carvalho Universitat Autònoma de Barcelona, Spain

Limit Cycles for Planar Systems with Invariant Algebraic Curves

Abstract. Our main goal in this work is to study the number of small-amplitude isolated periodic orbits, so-called limit cycles, that bifurcate from a single equilibrium point in a class of polynomial systems possessing an invariant algebraic curve.

David Rojas Universitat Autònoma de Barcelona, Spain

Monotonous period function for equivariant differential equations with homogeneous nonlinearities

Abstract. Real planar autonomous analytic differential systems can be written in complex coordinates in the compact form $\dot{z} = F(z, \overline{z})$, $z \in \mathbb{C}$. The $\mathbb{Z}_k$-equivariant differential equations are equations of this form that are invariant under a rotation through the angle $2\pi/k$ about the origin. For them, the phase portrait on each sector of width $2\pi/k$ centered at the origin is repeated k times. In this paper we are concerned with the simplest family of polynomial $\mathbb{Z}_k$-equivariant differential equations with a non-degenerated center at the origin and homogeneous nonlinearities

$$\dot{z} = iz + a(z\overline{z})^n z^{k+1},$$

with n and k a positive integers and $0 \neq a \in \mathbb{C}$. The qualitative properties of the period function associated to the origin was posed as the 16th open problem in [1]. In particular, it is conjectured that it is monotonous decreasing. In this talk we give a positive answer to such conjecture. We show this result as corollary of proving that the period function of the center at the origin of a sub-family of the reversible quadratic centers is monotonous decreasing as well.

The results presented in this talk can be found in [2].

References

[1] A. Gasull. "Some open problems in low dimensional dynamical systems." *SeMA Journal* 78 (2021) 233–269.

[2] A. Gasull and D. Rojas. "Monotonous period function for equivariant differential equations with homogeneous nonlinearities." *Mediterr. J. Math.* 22 (2025) 112.

Ana Livia Rodero Universidade de São Paulo, Brazil

Exploring the center and cyclicity problems of PSVF with cross-type switching region

Abstract. In this work, the center and the cyclicity problems inside a family of piecewise smooth vector fields (PSVF) with cross-type switching region was analyzed. We show 4 piecewise-smooth center families, all of them having generically at least cyclicity 4 up to the first order of Lyapunov constants. Besides, we provide a weak focus for which at least 7 small-amplitude crossing limit cycles bifurcate from the origin. The studied system was motivated by modeling ship maneuvers.

This is a joint work with Regilene Oliveira and Leonardo P. C. da Cruz.

Henryk Żołądek University of Warsaw, Poland

A qualitative approach to hydrodynamics

Abstract. I propose an apparently novel approach to hydrodynamic type evolution equations. I demonstrate (on an example) that the standard procedure of elimination of the pressure from the Navier–Stokes equations is qualitatively incorrect. Namely, the two conditions for the fluid velocity, zero divergence and vanishing at the boundary, are not compatible from the physical point of view. I propose other scenarios of elimination of the pressure in dimensions two and three. They rely upon splittings of the space of vector fields into two summands, one of which consists of divergence free vector fields. The solutions to the initial value problem for the Navier–Stokes system exist (under some smallness assumption), but are not unique. This provides a substantial contribution to one of the millennium problems. In the cases of Burgers equation and the reaction diffusion equation I show that, if the initial condition is sufficiently small (in a precise sense), then the solution exists for all positive times and is unique. On the other hand, there exist situations when the solutions either cease to be unique or blow up to infinity.

Victoriano Carmona Universidad de Sevilla, Spain

Isolated invariant closed curves in planar piecewise linear systems with no sliding regions

Abstract. We study planar piecewise linear differential systems with two zones separated by a straight line, without sliding regions. The flow crosses the separation line transversally, except possibly at one point. It is known that such systems can have at most one limit cycle, which, if it exists, is hyperbolic. Besides limit cycles, these systems may exhibit other isolated non-trivial invariant closed curves, formed by a homoclinic, two heteroclinic connections, or fold-fold connections. We prove the uniqueness of these curves and determine their stability as a simple function of the parameters of the system.

This talk is the result of research carried out jointly with Fernando Fernández-Sánchez and Douglas D. Novaes.

Isaac A. García Universitat de Lleida, Spain

STABILITY OF SOME MONODROMIC SINGULARITIES WITH TWO EDGES IN THE NEWTON DIAGRAM

Abstract. This work focuses on the study of monodromic singularities in planar analytic families of vector fields whose Newton diagram consists of exactly two edges. We begin by analyzing the desingularization scheme of a *minimal model* of polynomial vector fields, denoted by $\mathcal{X}$, which includes only the monomials corresponding to the vertices of the Newton diagram. We then extend this minimal model to the so-called *Brunella–Miari vector fields* $\mathcal{X} \subset \mathcal{X}^{[1]}$, incorporating all monomials associated with points lying on the edges of the Newton diagram. As a second extension, we consider vector fields $\mathcal{X}^{[1]} \subset \mathcal{X}^{[2]}$ that include higher-order terms corresponding to points located above the polygonal line in the Newton diagram. The key point of our approach is to preserve the desingularization geometry at each extension step. We provide explicit desingularization procedures, which enables the computation of the linear part of the return map Π in cases where the desingularized singularity is associated with a hyperbolic polycycle.

This is a joint work with Jaume Giné and Víctor Mañosa.

Dmitry Sinelshchikov Instituto Biofisika (UPV/EHU, CSIC),
University of the Basque Country, Leioa and
Ikerbasque, Basque Foundation for Science, Spain

ON THE INTEGRABILITY OF THE PROJECTIVE EQUATIONS

Abstract. In this talk we consider integrability of a family of second-order differential equations that is a projection of a geodesic flow of a two-dimensional (pseudo) Riemannian manifold. We develop two approaches for establishing integrability of the projective equations. The first one is based on finding solution of equivalence problems for the family of projective equations and its integrable subcases. The second one is connected with the classification of quasi-polynomial invariants for the projective equations. We introduce the notion of generalized Darboux first integrals for Hamiltonian systems for geodesics and demonstrate how these integrals are connected to the quasi-polynomial invariants of the projective equations. We construct several families of integrable Riemannian metrics with generalized Darboux first integrals, which at the particular values of the parameters degenerate into rational first integrals of arbitrary degree.

The talk is based on three recent works with Jaume Giné [1, 2, 3].

References

[1] J. Giné and D. Sinelshchikov, "On the geometric and analytical properties of the anharmonic oscillator." *Commun. Nonlinear Sci. Numer. Simul.* 131 (2024) 107875.

[2] J. Giné and D. Sinelshchikov, "Integrability of oscillators and transcendental invariant curves." *Qual. Theory Dyn. Syst.* 24 (2025) 26.

[3] J. Giné and D. Sinelshchikov, "Metrisable oscillators and (super)integrable two-dimensional metrics." *Preprint* (2025).

Antonio E. Teruel Universitat de les Illes Balears, Spain

On the displacement map around heteroclinic saddle connections in piecewise linear systems

Abstract. Using one-parameter families of piecewise linear differential systems that exhibit period annuli bounded by heteroclinic connections between two linear saddles, this talk examines the expression of the displacement map near the connection under generic perturbations within the piecewise linear family. In particular, we provide the expansion of the first Melnikov function in terms of the principal monomials, determine the number of monomials that are independent, and establish lower bounds for the number of limit cycles bifurcating from the connection. This analysis allows us to describe the behavior of the principal monomials as the parameter approaches values for which the saddles become strongly resonant (the hyperbolicity ratio equals one).

This is a joint work with Rafel Prohens and Joan Torregrosa.

Colin Christopher University of Plymouth, United Kingdom

Extending Liouvillian integrability

Abstract. When considering vector fields with multiple Liouvillian first integrals, it seems very natural to also consider integrals which can be obtained by quadratures along the leaves of previously defined first integrals. We explain how this concept can be made algebraic and give some preliminary results in the case of three-dimensional vector fields.

This work described is part of an ongoing project in collaboration with Chara Pantazi, Daniel Robertz, and Sebastian Walcher.

Adriana Buică Universitatea Babeş-Bolyai, Romania

Discrete Malkin bifurcation functions

Abstract. We are interested in the persistence of ω-periodic solutions in non-autonomous ω-periodic difference systems in arbitrary finite dimension. We construct the so-called discrete Malkin bifurcation functions, showing an analogy with the continuous case. Our results complement the ones obtained by Pötzsche in 2013 or Gasull–Valls in 2022.

Douglas D. Novaes Universidade Estadual de Campinas, Brazil

Growth estimate for the number of crossing limit cycles in planar piecewise polynomial vector fields

Abstract. Motivated by the classical Hilbert's Sixteenth Problem, we investigate the growth of the maximum number of crossing limit cycles, denoted by $\mathcal{H}_c(n)$, in piecewise polynomial planar vector fields of degree n. The best known general lower bound for $\mathcal{H}_c(n)$ is $2n-1$. We adapt the recursive construction of Christopher and Lloyd to show that, within the piecewise polynomial Hamiltonian framework, $\mathcal{H}_c(n)$ grows at least as fast as $n \log n$. Moreover, in the more general setting of piecewise polynomial systems, we prove that $\mathcal{H}_c(n)$ grows at least as fast as n^2, thus improving the previously known linear asymptotic behavior. Inspired by recent results, we also establish that $\mathcal{H}_c(n)$ is a strictly increasing function whenever it is finite, and that in such cases it can be realized by piecewise polynomial vector fields possessing only hyperbolic crossing limit cycles. These results extend to discontinuous systems the advances previously obtained for the Hilbert number in smooth polynomial vector fields.

This work is being developed in collaboration with Luana Ascoli.

Enrique Ponce Universidad de Sevilla, Spain

Reversible periodic linear systems: the planar case

Abstract. The existence of a time-reversal symmetry in periodic linear systems imposes some structure in the monodromy matrix. For planar systems, we study the local geometry of their stability boundaries around some critical values of parameters. Two different instances of Hill's equation will be addressed.

This is a joint work with Emilio Freire and Manuel Ordóñez.

Josef Hofbauer University of Vienna, Austria

Planar S-systems

Abstract. Planar S-systems are ODEs of the form $\dot{x} = p_1 - p_2, \dot{y} = p_3 - p_4$ with $p_i = c_i x^{a_i} y^{b_i}$ and $a_i, b_i, c_i > 0$, defined on the positive quadrant $\mathbb{R}^2_+$. I will discuss limit cycles, the center problem, and boundedness of solutions for this class of planar systems.

References

[1] B. Boros and J. Hofbauer. "Planar S-systems: Permanence." *J. Diff. Equations* 266 (2019) 3787–3817.

Daniel Panazzolo Université de Haute-Alsace, France

Regularization by convolution and smoothing of piecewise smooth vector fields

Abstract. We study a general regularization procedure for piecewise smooth vector fields whose discontinuity locus is a variety of normal crossings type. We show that such regularization can be smoothed through a finite sequence of blowings-up, thereby reducing the problem to study of the dynamics of a smooth vector field in a manifold with corners. I will illustrate the procedure in the specific case of a piecewise constant planar vector field in $\mathbb{R}^2$ with discontinuity locus $xy = 0$. In this case, we generically obtain a Bogdanov–Takens bifurcation in the blowing-up locus.

This is a joint work with Claudio Buzzi and Paulo R. Silva.

Hans-Christian v. Bothmer University of Hamburg, Germany

Darboux integrability via singularities of integral curves

Abstract. I will explain how singularities of plane integral curves can be used to prove the existence of a rational integrating factor from the geometry of the integral curves alone. As an application one can construct new codimension 11 components of the center variety in degree 3.

Luis Fernando Mello Universidade Federal de Itajubá, Brazil

Injectivity of maps in the plane

Abstract. Let $F : \mathbb{R}^2 \to \mathbb{R}^2$ be a C^2 map with a non-zero Jacobian determinant. By the Inverse Function Theorem this map is locally injective. A classical and difficult problem is to know when this map is globally injective – a global diffeomorphism if furthermore F is surjective. By using results of the qualitative theory of differential equations, I will present a sufficient and necessary condition and a sufficient condition for the global injectivity of F.

The presentation is based on studies developed with Jaume Llibre.

Jaume Llibre Universitat Autònoma de Barcelona, Spain

Invariant algebraic curves and rational first integrals for planar polynomial vector fields

Abstract. We present three main results. The first two provide sufficient conditions in order that a planar polynomial vector field in $\mathbb{C}^2$ has a rational first integral, and the third one studies the number of multiple points that an invariant algebraic curve of degree n of a planar polynomial vector field of degree m can have in function of m and n.

These results were obtained together with Javier Chavarriga.

Xiang Zhang Shanghai Jiao-Tong University, China

Reduction of elementary integrability of polynomial vector fields

Abstract. Prelle and Singer proved in 1983 that if a system of n ordinary differential equations defined on a differential field K has a first integral in an elementary field extension L of K, then it must have a first integral consisting of algebraic elements over K via their constant powers and logarithms. Based on this result they further provided a reduction on elementary integrable planar polynomial differential systems having an integrating factor which is a fractional power of a rational function. Here we extend their result and obtain that if the ordinary differential system has $\ell < n$ functionally independent first integrals in an elementary field extension L of K, then it must have ℓ functionally independent first integrals consisting of algebraic elements over K via their constant powers and logarithms. Moreover, applying our established reduction result, we can further verify that any finite dimensional elementary integrable polynomial differential system has a rational Jacobian multiplier.

Peter de Maesschalck Hasselt University, Belgium

Slow-fast solutions of Abel equations

Abstract. We explore trigonometric Abel equations with large coefficients, or equivalently, slow-fast Abel equations. Using geometric singular perturbation theory, we prove the possibility of canard-type periodic solutions, as well as a solution type that we call canard spike solutions.

This is a joint work with María J. Álvarez.

Joan C. Artés Universitat Autònoma de Barcelona, Spain

Topological invariants applied to the classification of phase portraits of quadratic differential systems. The birth of an Encyclopedia on quadratic systems

Abstract. Along the years, there have appeared many classifications of phase portraits of quadratic differential systems. Some having some few portraits and others with several hundreds. At this moment, it is not yet known the number of topologically distinct phase portraits of quadratic systems, not even modulo limit cycles. Even more, it is not always trivial to realize if two phase portraits are topologically distinct or not, specially when one needs to compare many phase portraits, even within one same paper. And often, an author (even myself) has assured that all the phase portraits he presents are topologically distinct, and they are not. I will present some topological invariants which work to describe every phase portrait of a quadratic differential system, and which serves to compare in a systematic way all the phase portraits of a paper, and also to compare phase portraits of different papers. This method has been applied to a set of 80 papers containing altogether more than 4500 phase portraits and allows to detect easily all the papers in which a same phase portrait has appeared. Moreover, it serves to detect wrong phase portraits and missed ones. This is also very useful for any new classification since it may easily detect if some phase portrait is missing in it, or if it brings up a still unknown phase portrait.

Armengol Gasull Universitat Autònoma de Barcelona, Spain

Riccati differential equations and limit cycles

Abstract. From many points of view, Riccati differential equations are well understood. For instance, their time-T flow is a Möbius map, which implies that they can have at most two isolated periodic solutions (limit cycles). The aim of this talk is to show that these equations still present interesting open questions and, perhaps more importantly, that they arise naturally in the study of the exact number of limit cycles for several families of planar differential equations. As an illustration, we describe some results on the number of limit cycles in certain families of rigid systems [2] and piecewise-rigid systems [3]. We also present new results concerning the exact number of limit cycles for several families of Riccati equations [1].

References

[1] A. Gasull, D. D. Novaes, and J. Torregrosa. "Weak-Coppel problem for a class of Riccati differential equations." *Preprint* (2025).

[2] A. Gasull and J. Torregrosa. "Exact number of limit cycles for a family of rigid systems." *Proc. Amer. Math. Soc.* 133 (2005) 751–758.

[3] A. Gasull and J. Torregrosa. "Limit cycles for piecewise rigid systems with homogeneous non-linearities." *In preparation.*

Poster session

Eduarda D. de Almeida Université de Haute-Alsace, France

O-minimal Theory: A tool to study the finiteness of limit cycles of piecewise vector fields

Abstract. The second part of Hilbert's sixteenth problem has inspired many significant advances in the qualitative theory of differential equations [1]. Motivated by this classical problem, we study its analogue in the setting of piecewise vector fields. Given a collection of r horizontal and s vertical lines, consider the partition $\mathcal{P}_{(r,s)}$ of the plane into $k(r,s) = 4rs + 2(r+s) + 1$ disjoint cells,

$$\mathbb{R}^2 = \bigsqcup_{i=1}^{k(r,s)} C_i,$$

where each cell is of dimension 0(points), 1(segments), or 2(rectangles). Let $PC^2(\mathcal{P}_{(r,s)})$ denote the space of piecewise constant vector fields associated with this partition. We will explain the proof of the following result:

Theorem. *There exists a natural number $N(r,s)$ such that each vector field in $PC^2(\mathcal{P}_{(r,s)})$ has at most $N(r,s)$ semi-limit cycles.*

This theorem is a first step towards the conjecture stated in [2] for the more general class $PL^2(\mathcal{P}_{(r,s)})$ of piecewise linear vector fields over a partition $\mathcal{P}_{(r,s)}$, and our approach extends the result of [3]. For this purpose, the main tool we used was the o-minimal theory [4], which allowed us to obtain the finiteness of the limit cycles in this setting.

References

[1] Y. Ilyashenko, "Centennial history of Hilbert's 16th problem," *Bulletin of the American Mathematical Society*, vol. 39, no. 3, pp. 301–354, 2002.

[2] D. Panazzolo, "Tame semiflows for piecewise linear vector fields," *Annales de l'institut Fourier*, vol. 52, no. 6, pp. 1593–1628, 2002.

[3] V. Carmona, F. Fernández-Sánchez, and D. D. Novaes, "Uniform upper bound for the number of limit cycles of planar piecewise linear differential systems with two zones separated by a straight line," *Applied Mathematics Letters*, vol. 137, p. 108501, 2023.

[4] L. Van den Dries, *Tame topology and o-minimal structures*, vol. 248. Cambridge university press, 1998.

Paulina Mancilla-Martínez Universidad del Bío-Bío, Chile.

On the Hamiltonian Hopf bifurcation in the 3D Hénon–Heiles Hamiltonian revised

Abstract. In 2002 it was conjectured that the Hamiltonian system of the 3D Hénon-Heiles Hamiltonian

$$H(p,q) = \frac{1}{2}\left(p_1^2 + p_2^2 + p_3^2\right) + \frac{1}{2}\left(q_1^2 + q_2^2 + q_3^2\right) + a\left(\lambda q_3\left(q_1^2 + q_2^2\right) + \frac{q_3^3}{3}\right),$$

can exhibit a Hamiltonian Hopf bifurcation for the λ values of $1/2$ and $5/2$. This conjecture was proved almost immediately for the values of $\lambda = 1/2$ and also for λ near $5/2$ and -1, when the parameter a is sufficient small. In this work we show that this Hamiltonian system exhibits a Hamiltonian Hopf bifurcation for the values of $\lambda \in (-\infty, -1] \cup [1/2, 1] \cup [5/2, \infty)$ and for all value of parameter a. Moreover, we provide analytical approximation of the three periodic orbits bifurcating from the Hamiltonian Hopf equilibrium at the origin of the Hamiltonian system for these values of the parameters λ and a.

This is a joint work with Jaume Llibre.

References

[1] S. Ferrer, H. Hanβmann, J. Palacián, and P. Yanguas. "On perturbed oscillators in 1-1-1 resonance: the case of axially symmetric cubic potentials." *J. Geom. Phys* 40 (2002), 320–369.

[2] H. Hanβmann and J. C. Van der Meer. "On the Hamiltonian Hopf bifurcations in the 3D Hénon-Heiles family.", *J. Dynamics and Differential Equations* 14 (2002), 675–695.

[3] M. Hénon and C. Heiles. "The applicability of the third integral of motion: Some numerical experiments." *Astron. J.* 69 (1964), 73–79.

Pedro C. C. R. Pereira Universidade Estadual de Campinas, Brazil

Integral tori in non-autonomous planar differential equations via averaging theory

Abstract. A classical and useful result from the averaging theory relates the existence of isolated periodic solutions of non-autonomous periodic differential equations with the existence of simple singularities of the so-called associated guiding system, which is an autonomous differential equation given in terms of the first non-vanishing higher order averaged function. We will discuss an extension of this result: that hyperbolic periodic orbits in the phase of the guiding system guarantee the existence of integral tori for the original non-autonomous differential equation. Regularity, convergence, and stability of such tori, as well as the dynamics displayed on them, are also investigated.

References

[1] D. D. Novaes and P. C. C. R. Pereira. "Invariant tori via higher order averaging method: existence, regularity, convergence, stability, and dynamics." *Mathematische Annalen* (2023).

[2] P. C. C. R. Pereira, D. D. Novaes, and M. R. Cândido. "A mechanism for detecting normally hyperbolic invariant tori in differential equations." *J. Math. Pures Appl.* 177 (2023) 1–45.

Set Pérez-González Universidad de Oviedo, Spain

Refining the Dynamic Modeling of Rocking Motion

Abstract. Housner's pioneering study [1] established the dynamics of a rigid block subjected to an external impulse, revealing that Rocking Motion (RM) –despite its seemingly simple setup– can exhibit remarkably complex behavior. This complexity stems from the discontinuous transitions between rest and oscillation about different pivot points, as well as from the intermittent impacts with the supporting surface. More recently, [2]

introduced a framework that reformulated the classical piecewise RM equations as a continuous system of differential equations and reinterpreted impact forces –originally modeled through a restitution coefficient– as a coupling mechanism generating instantaneous Dirac-type impulses. This regularized formulation also enables the study of broader classes of dynamical systems. Starting from the system of central forces that describes Rocking Motion, where angular momentum is a conserved quantity, in our work we managed to generalize it into a singular system that admits a certain polynomial regularization. This allows to analyze the complete dynamics of the system and its corresponding 2-parameter bifurcation diagram.

This is a joint work with Jesús S. Pérez del Río and Francisco Prieto.

References

[1] G. Housner. "The behavior of inverted pendulum structures during earthquakes." *Bull. Seismol Soc. Am.* 53 (1963), 403–417.

[2] F. Prieto and P. B. Lourenço. "On the rocking behavior of rigid objects." *Meccanica* 40 (2005), 121–133.

Lucas Queiroz Universidade Estadual de Campinas, Brazil

New lower bounds for limit tori in 3D polynomial vector fields

Abstract. We investigate the maximal number $N_h(m)$ of normally hyperbolic limit tori in three-dimensional polynomial vector fields of degree m, which extends the classical notion of Hilbert numbers to higher dimensions. Using recent developments in averaging theory, we show the existence of families of vector fields near monodromic singularities, including both Hopf-zero and nilpotent-zero cases, that exhibit multiple nested normally hyperbolic limit tori. As a result, we obtain new lower bounds on $N_h(m)$.

Roberto Trinidad-Forte Universitat de les Illes Balears, Spain

Global centers in piecewise linear equations in the cylinder

Abstract. We characterize global centers (all solutions are periodic) of the piecewise linear equation $x' = a(t)|x| + b(t)$ when the coefficients a, b are trigonometric polynomials, under some generic hypotheses. We prove that the global centers are those determined by the composition condition on a, b. That is, the equation has a global center if and only if there exist polynomials P, Q and a trigonometric polynomial h such that $a(t) = P(h(t))h'(t)$, $b(t) = Q(h(t))h'(t)$.
