

# **The three million body problem**

Don Saari  
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**Wonderful birthday celebration!!**

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**Wonderful birthday celebration!!**

But, “No good deed goes unpunished”

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Intention:

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Encourage the use of dynamics to better understand world/universe

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And, I was asked to discuss this by a particular committee

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No, still only **3,000,000**

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Current number of Jaume Llibre's coauthors

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**Reality constraint on  
my lecture!**

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Happy birthday Jaume!!!



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Emphasis on importance of problem, on how we can help

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## Space junk

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over 1100 satellites that need to be protected

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about 3,000,000 pieces of junk trying to collide

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## Beer Cans, Cigarettes In Space, Attributed To Astronauts

By [David M. Kruk](#)

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Man-made rubbish

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Man-made rubbish  
litter such as nuts, bolts, but also

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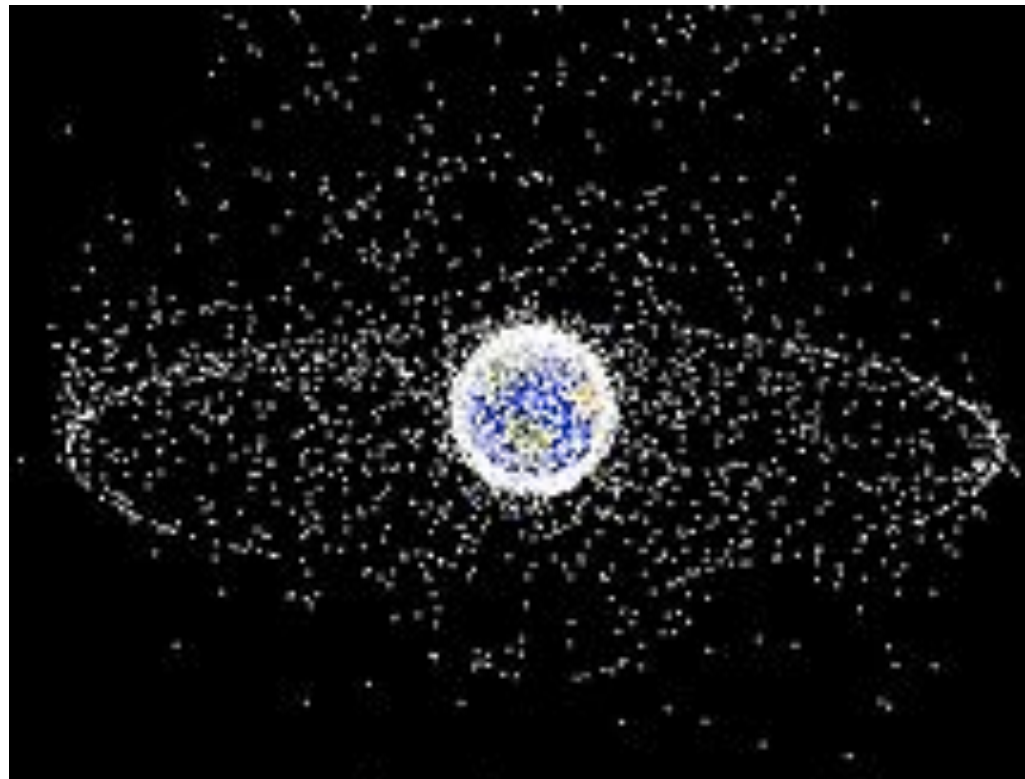
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## We must

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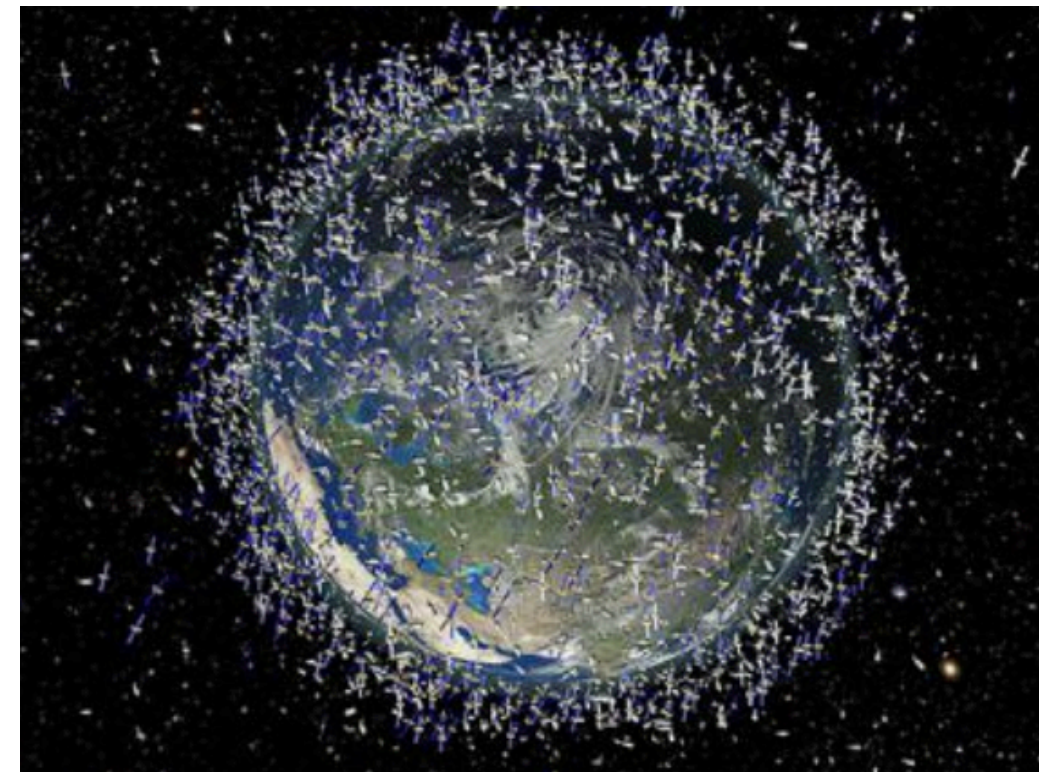
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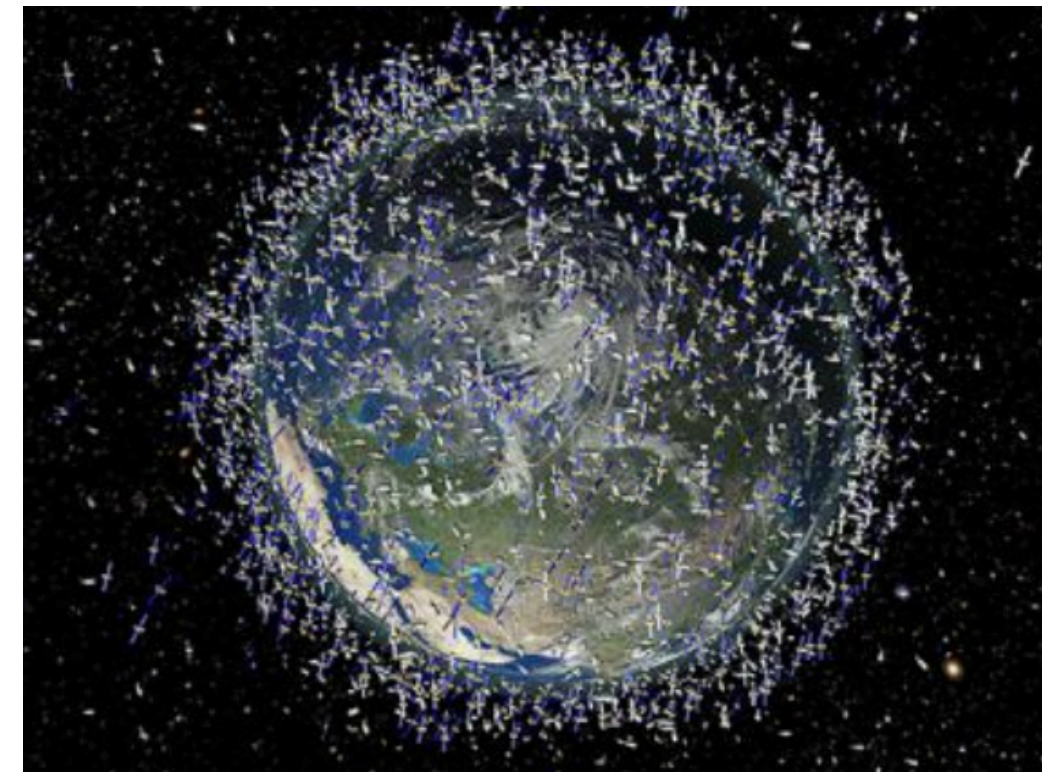
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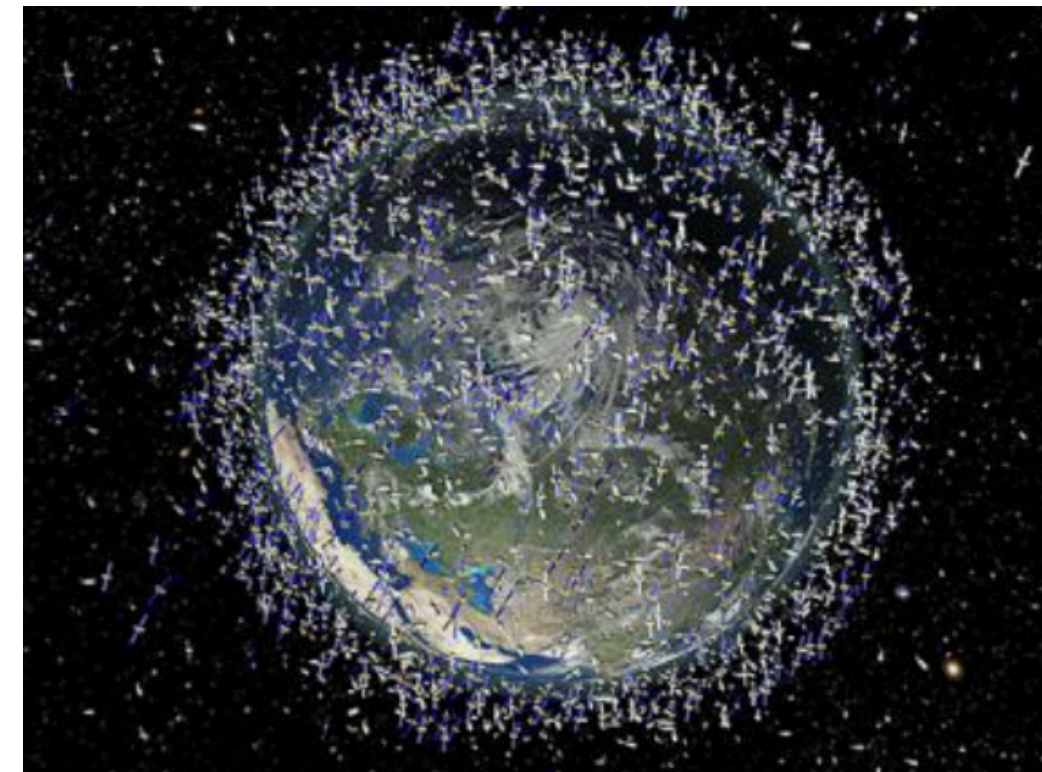
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Affect satellite communication systems  
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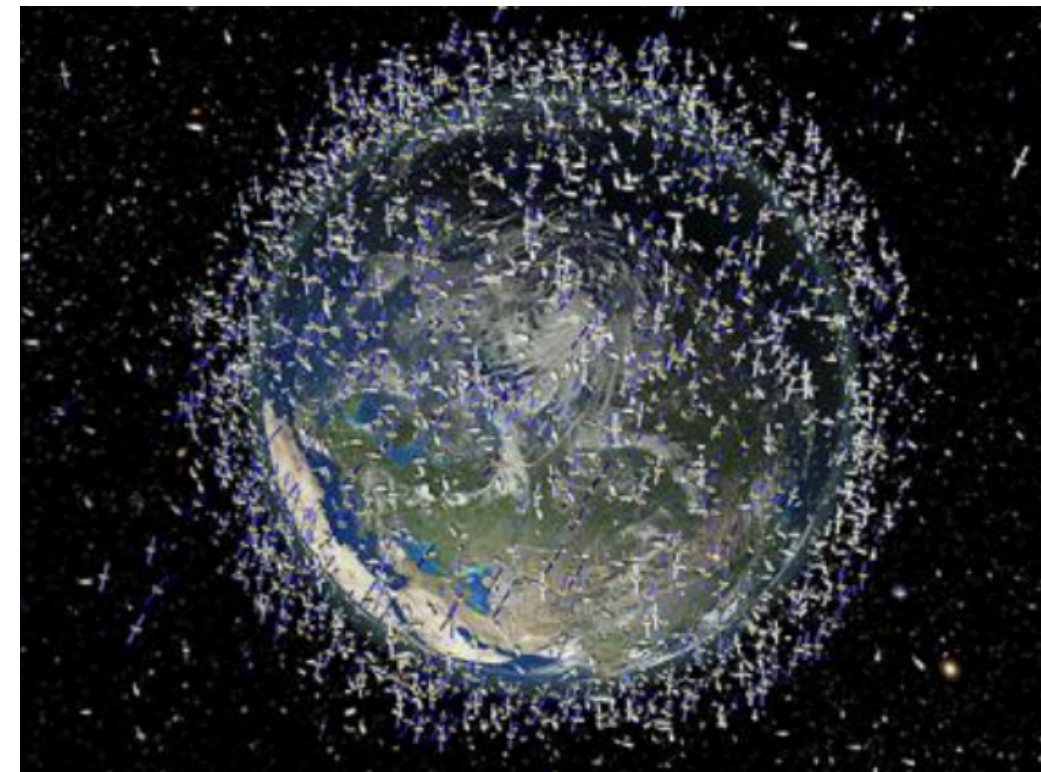
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Space so full of junk that a satellite collision could destroy communications on Earth

Space is so littered with debris that a collision between satellites could set off an “uncontrolled chain reaction” capable of destroying the communications network on Earth, a Pentagon report warned.



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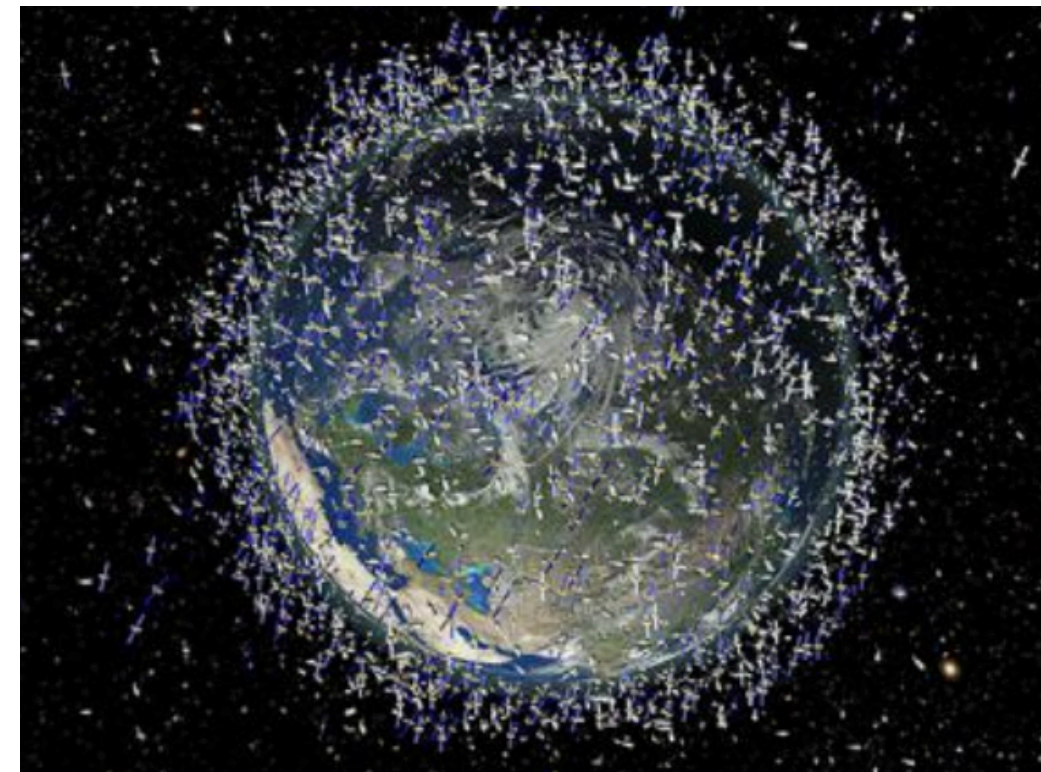
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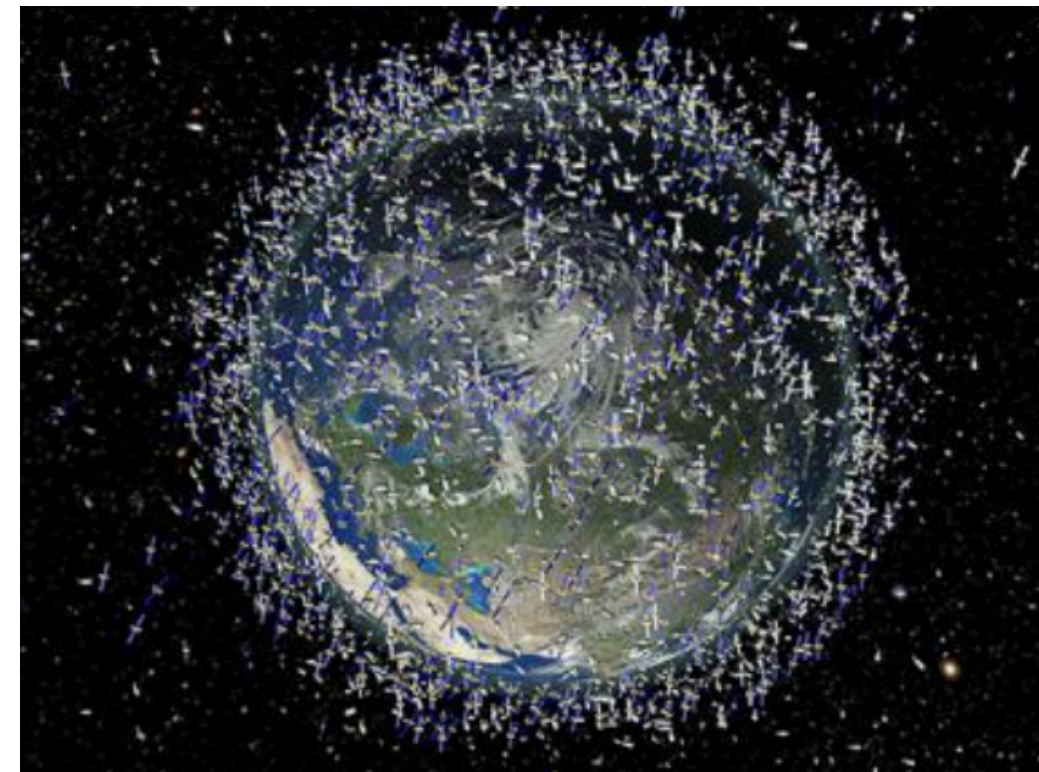
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2011 NRC report: Amount of debris at critical level

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windows of space probes

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12 - 13 km/second



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size of bb from Daisy bb gun

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Communication satellite



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Communication satellite  
ISS has shield -- Whipple Bumper



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The ISS seen from Space Shuttle *Endeavour* on May 30,  
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Third time in history needed to be done  
Also, need to avoid collisions; e.g. on  
April 1, 2011, fired thrusters to skirt  
debris

# Collisions

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A replica of an Iridium satellite.

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3 inches: must move out of the way



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Clearly, need to predict

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Remember: bumpers, but usually up to 1 cm. (Even 1 to 10 cm objects can be tracked, but difficult!) If piece is 1 kg (10 cm); will destroy 1,000 kg spacecraft

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# Analysis



Analysis

Commercial, government

Analysis      Commercial, government  
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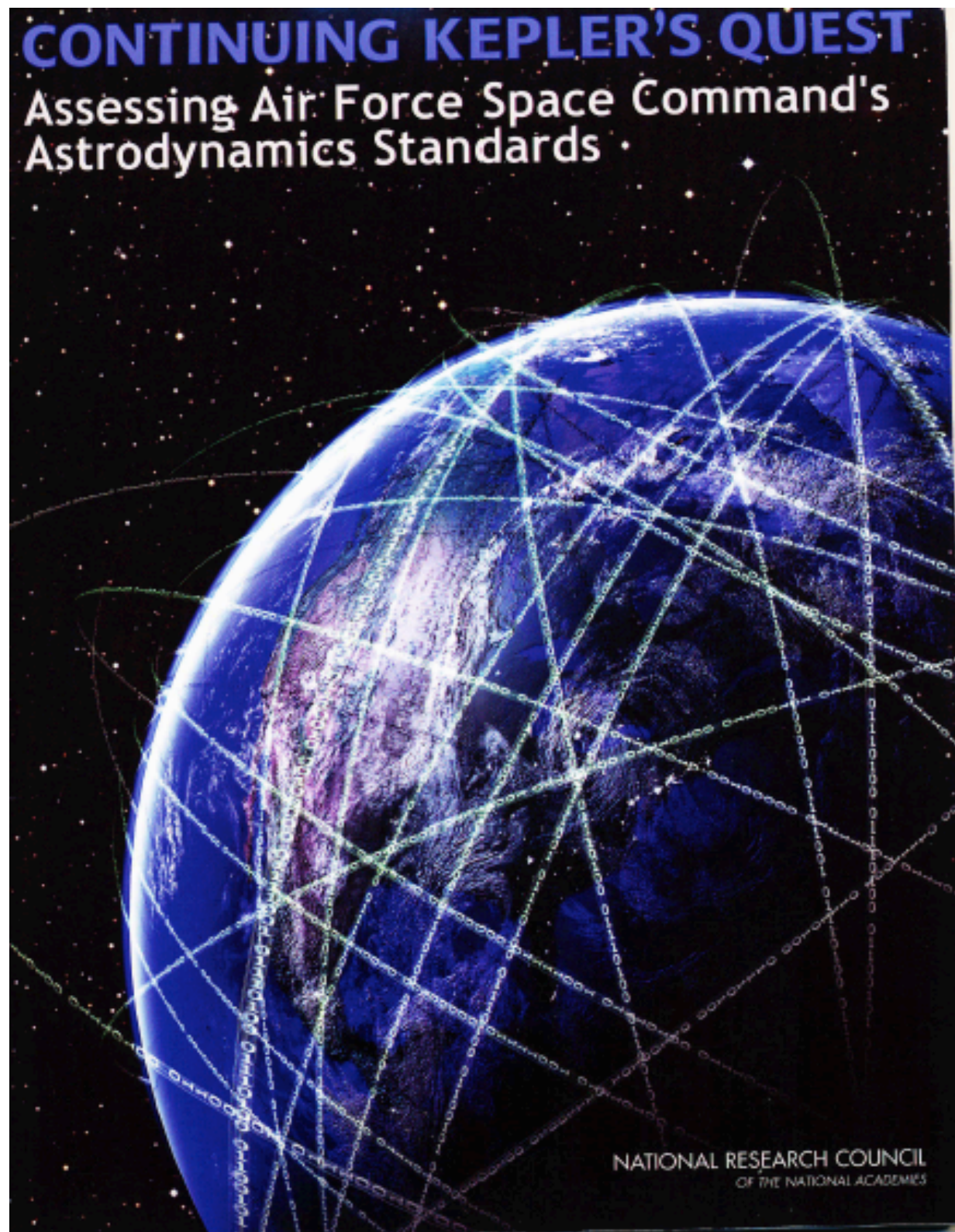
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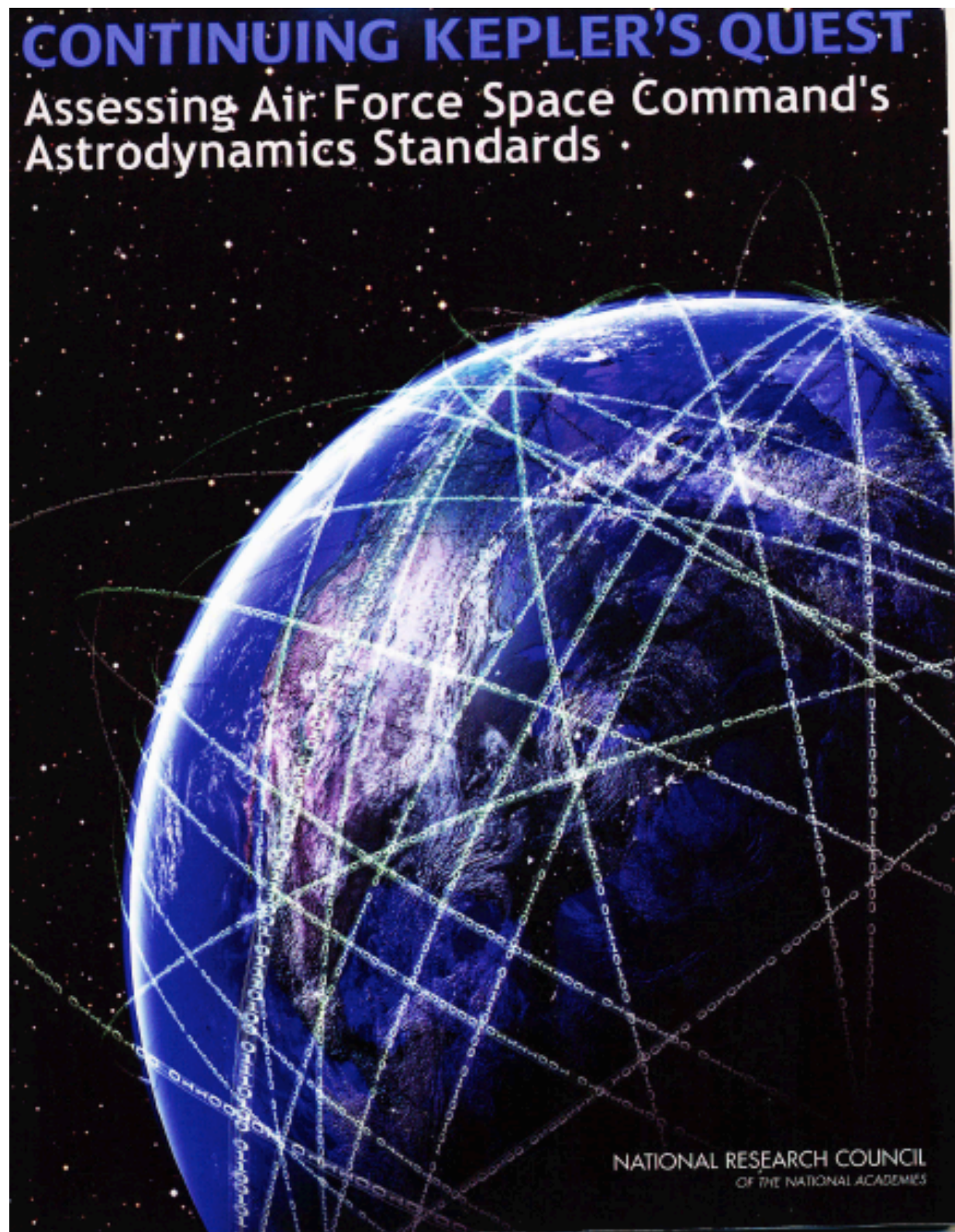
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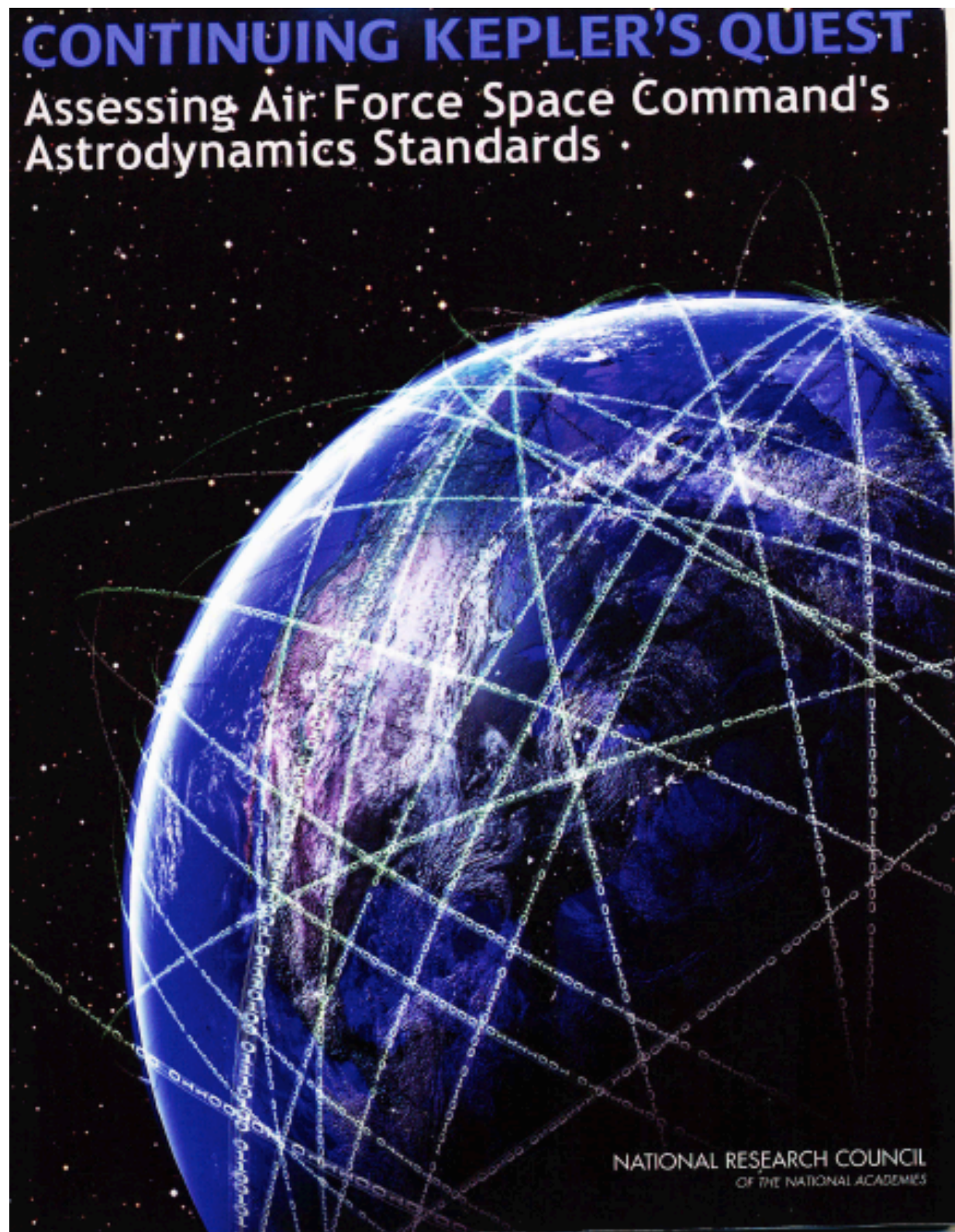
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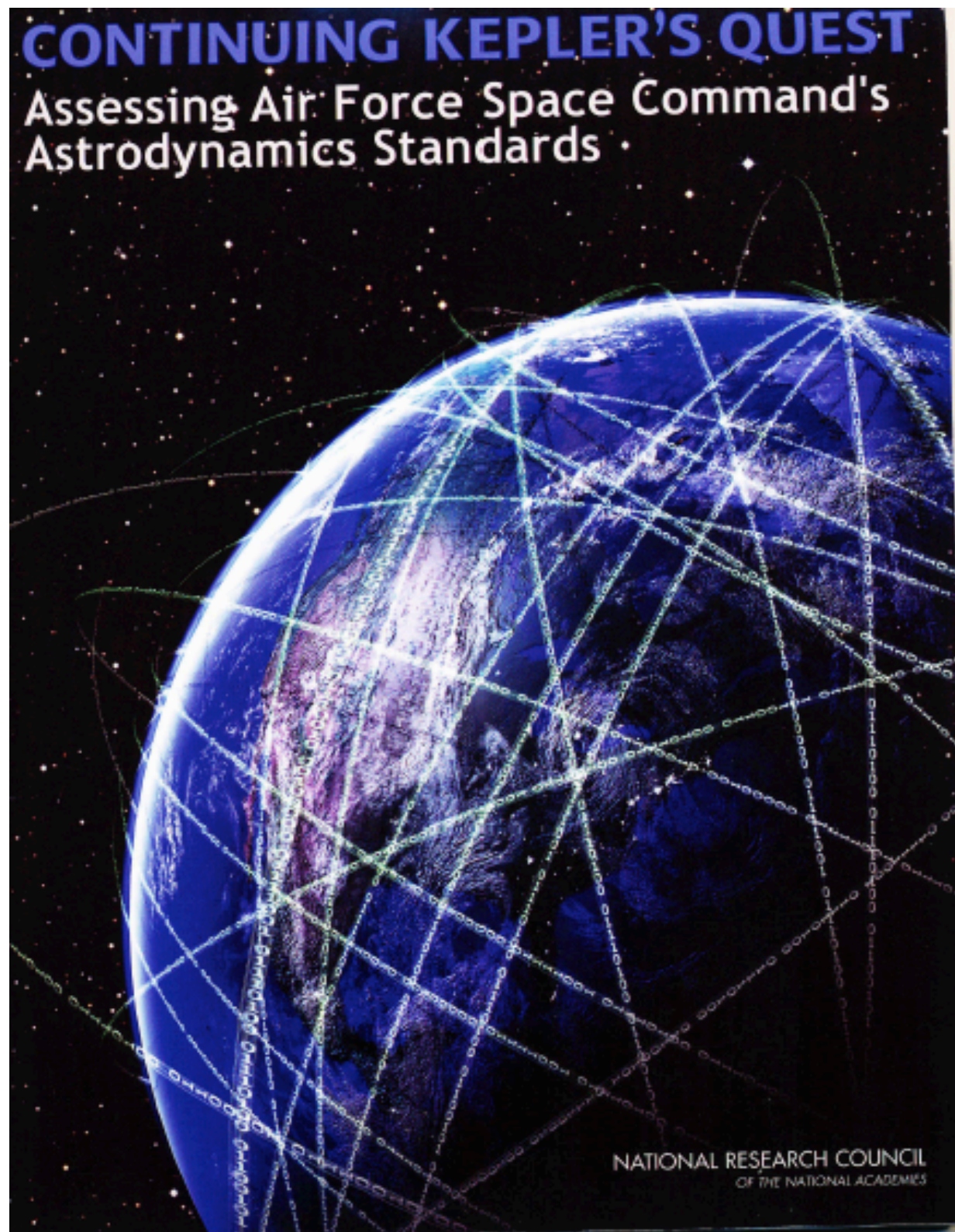
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send me an email; [dsaari@uci.edu](mailto:dsaari@uci.edu)





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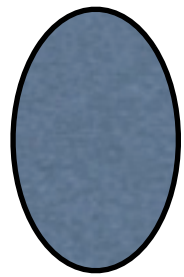
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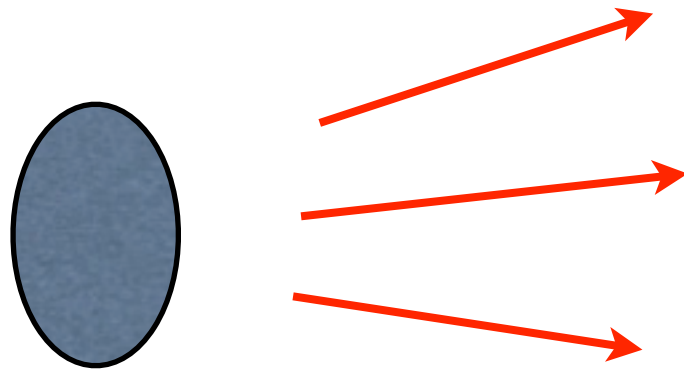
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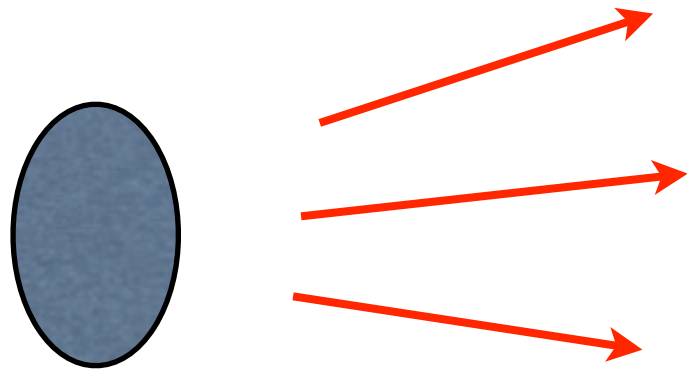


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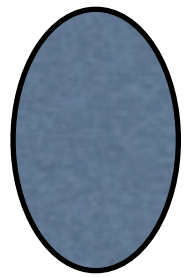
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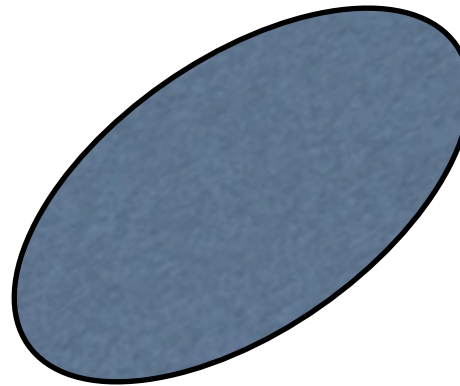
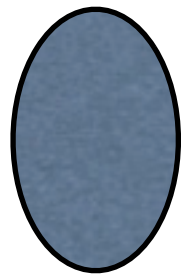
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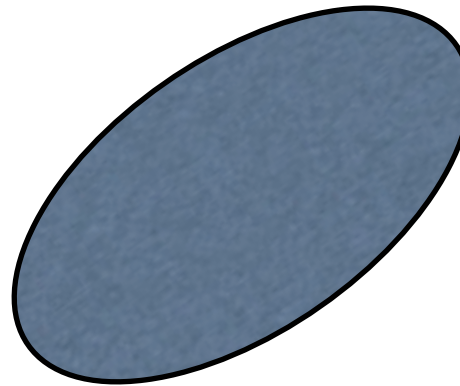
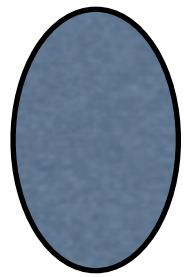
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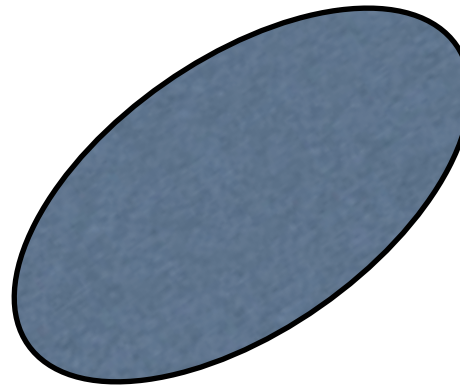
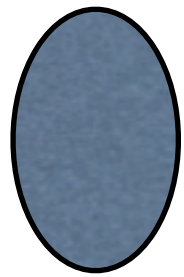
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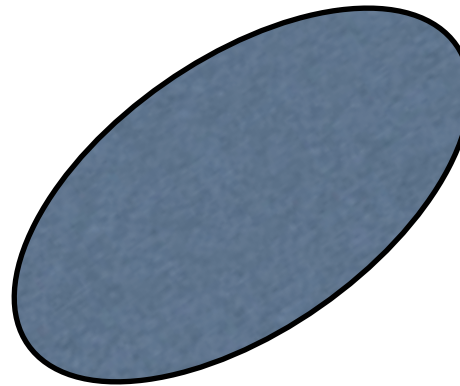
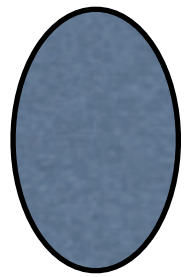
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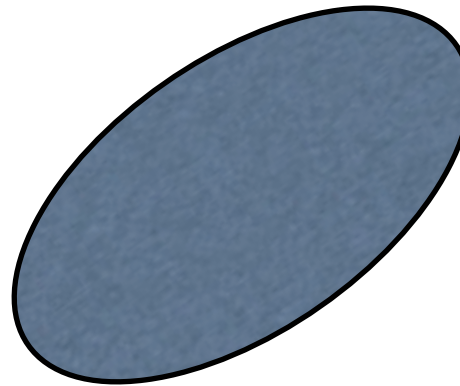
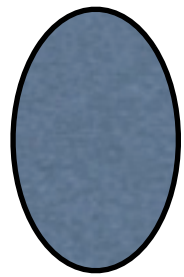
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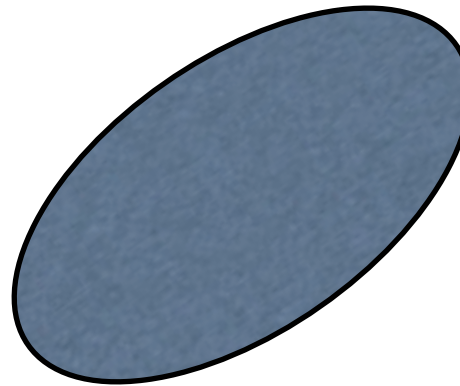
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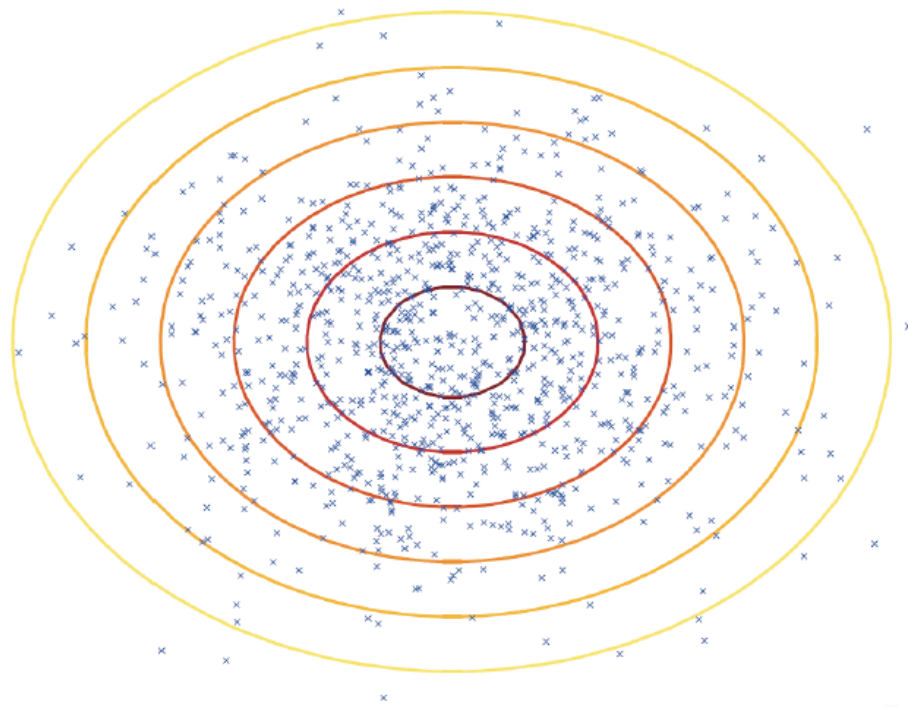
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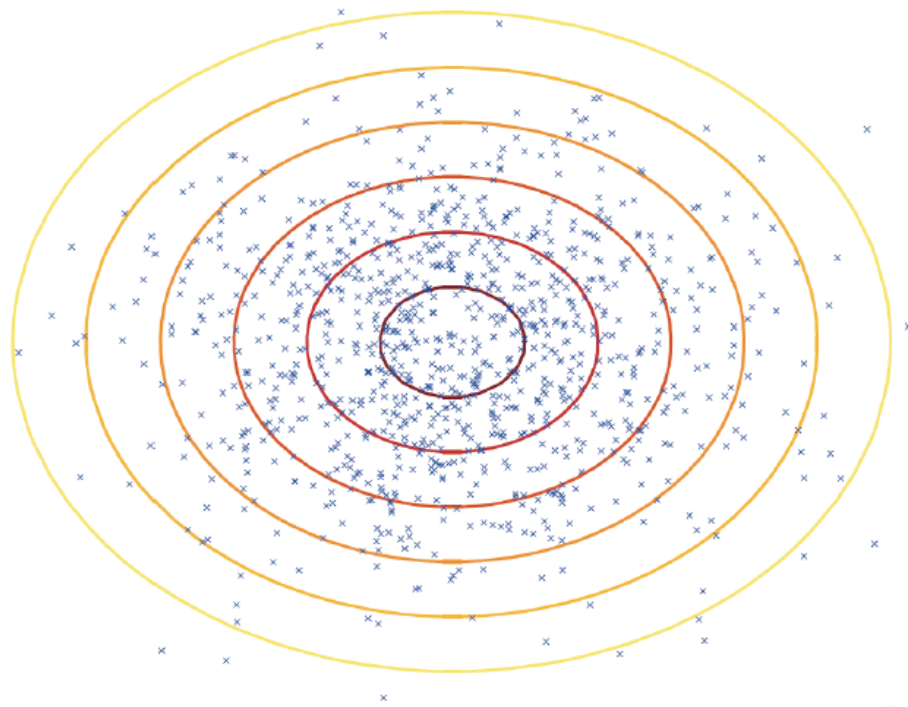
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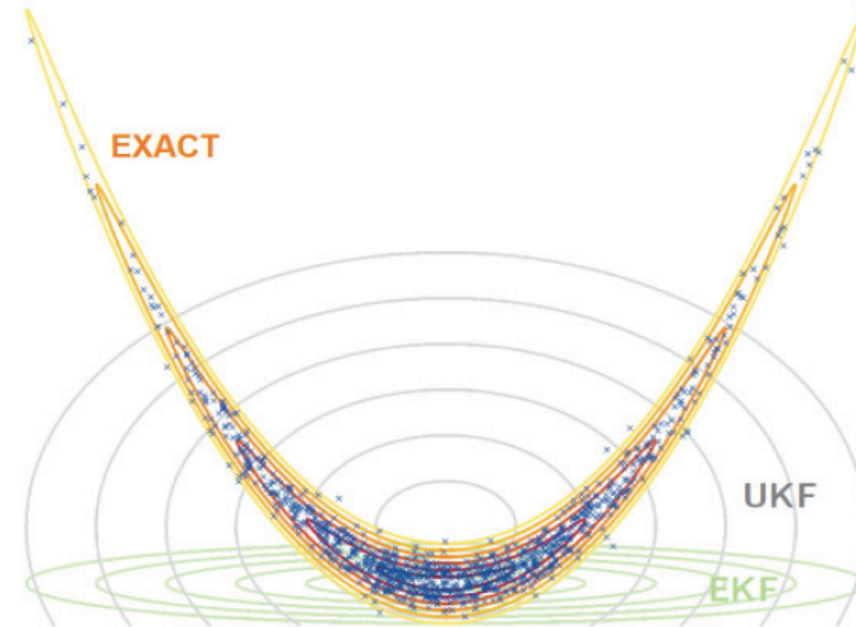
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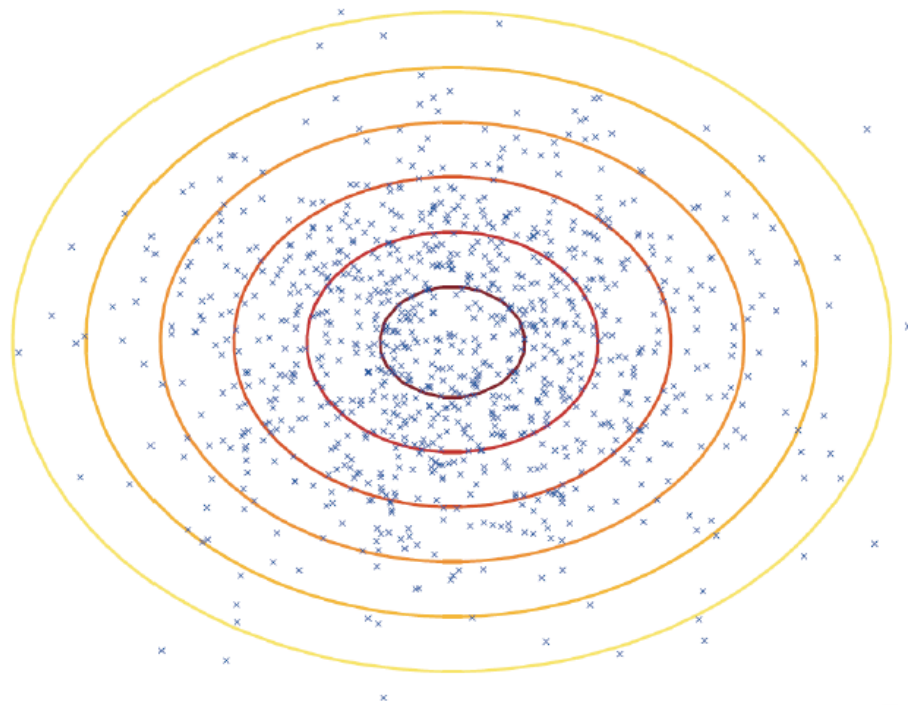


Prediction

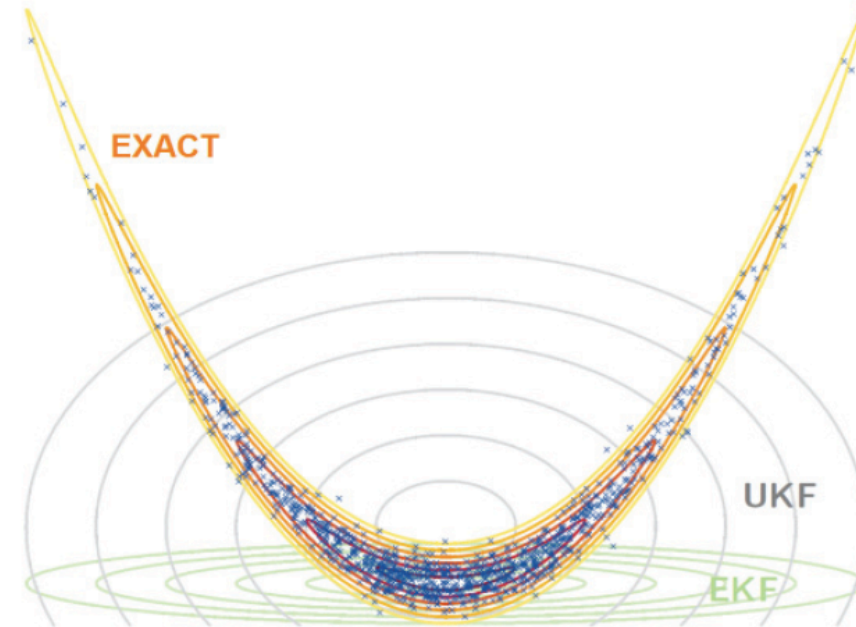


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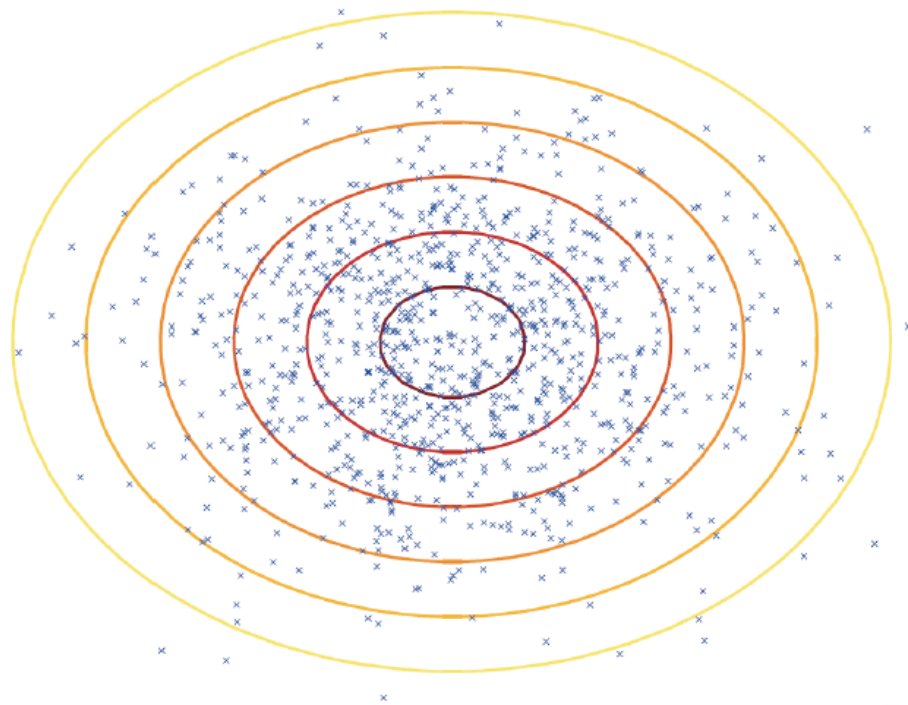


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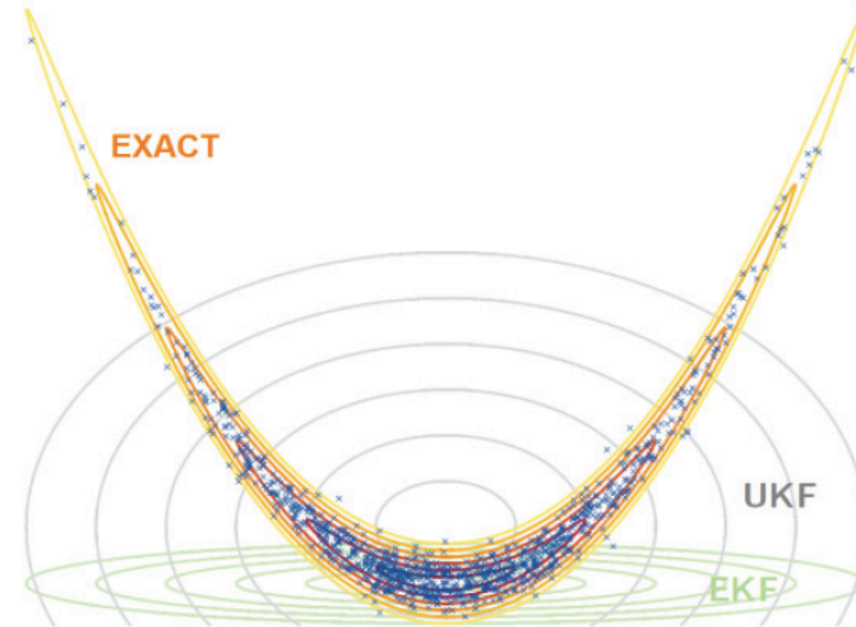


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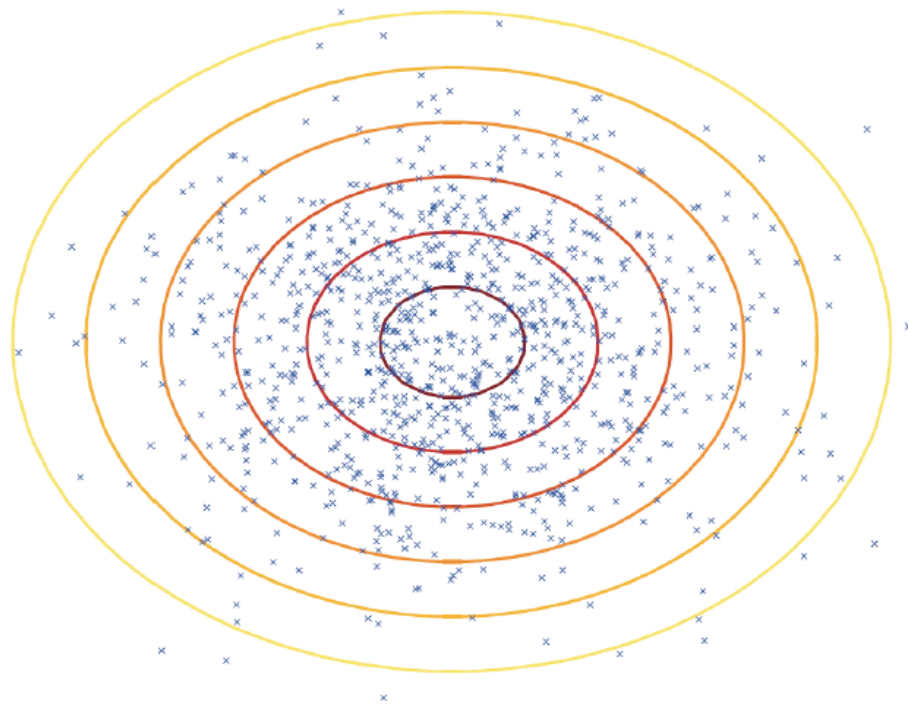


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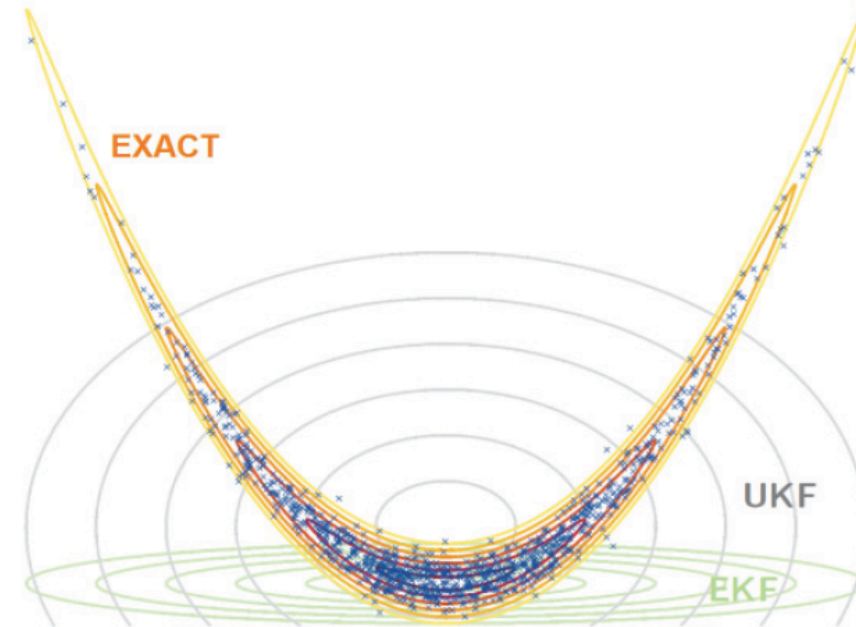


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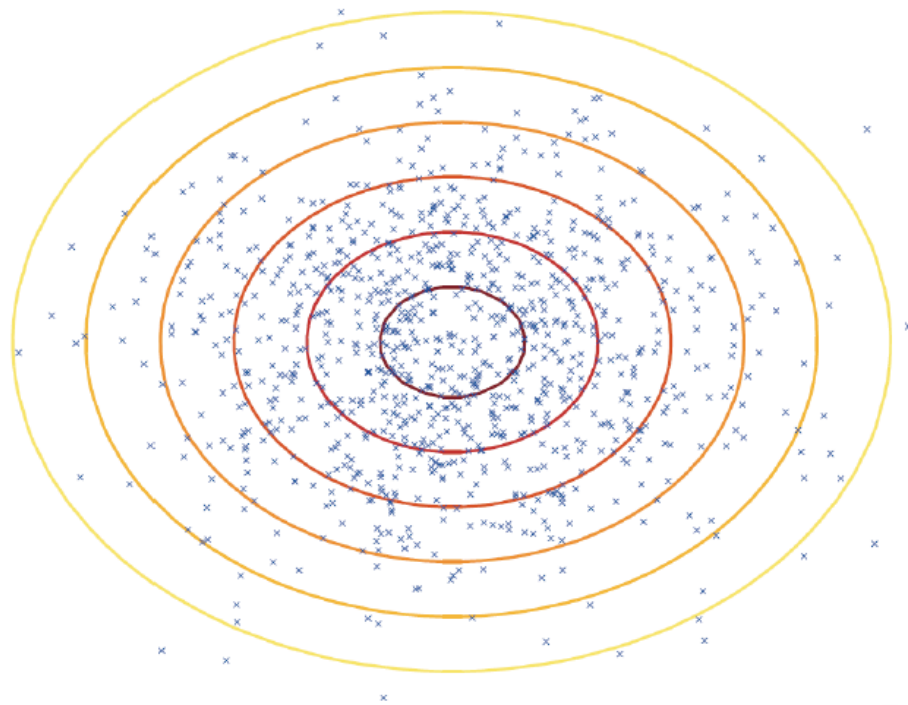


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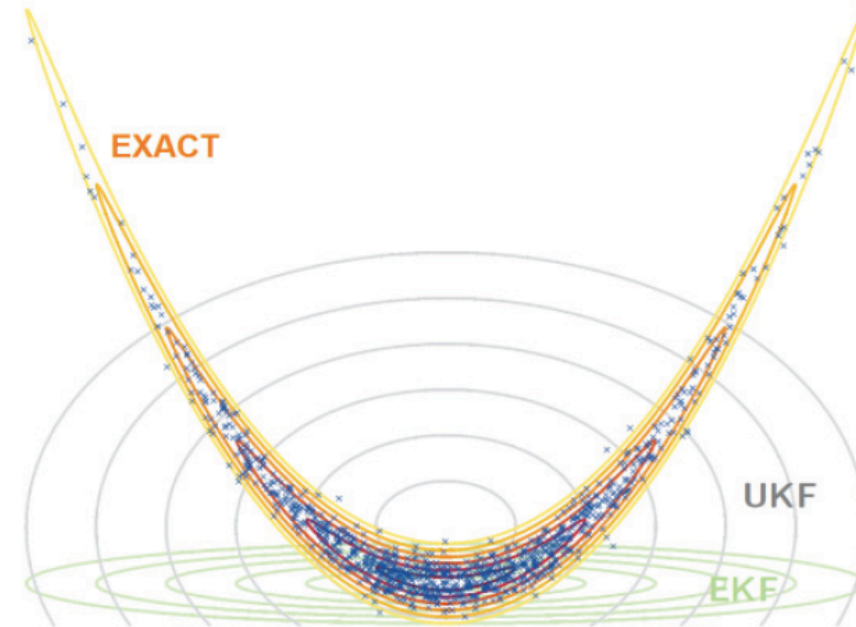


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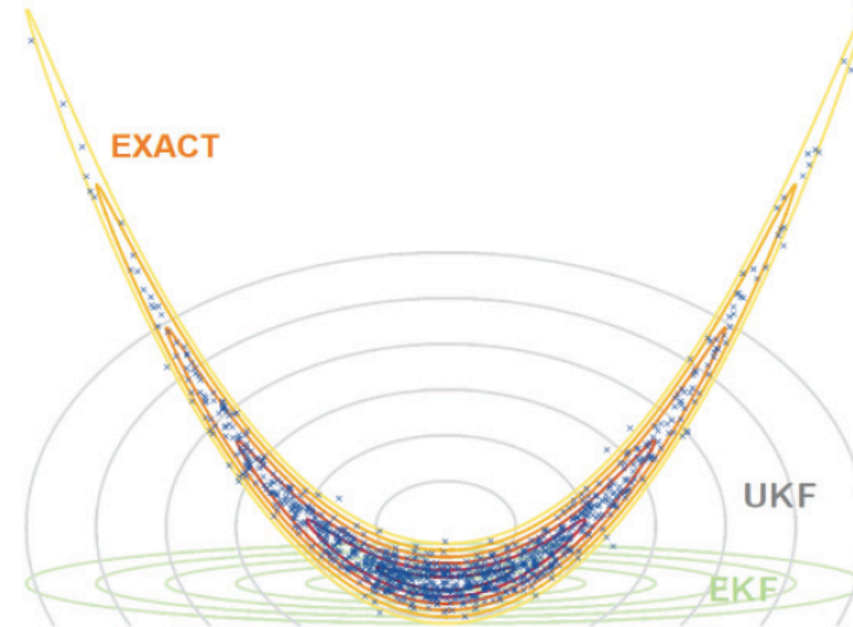
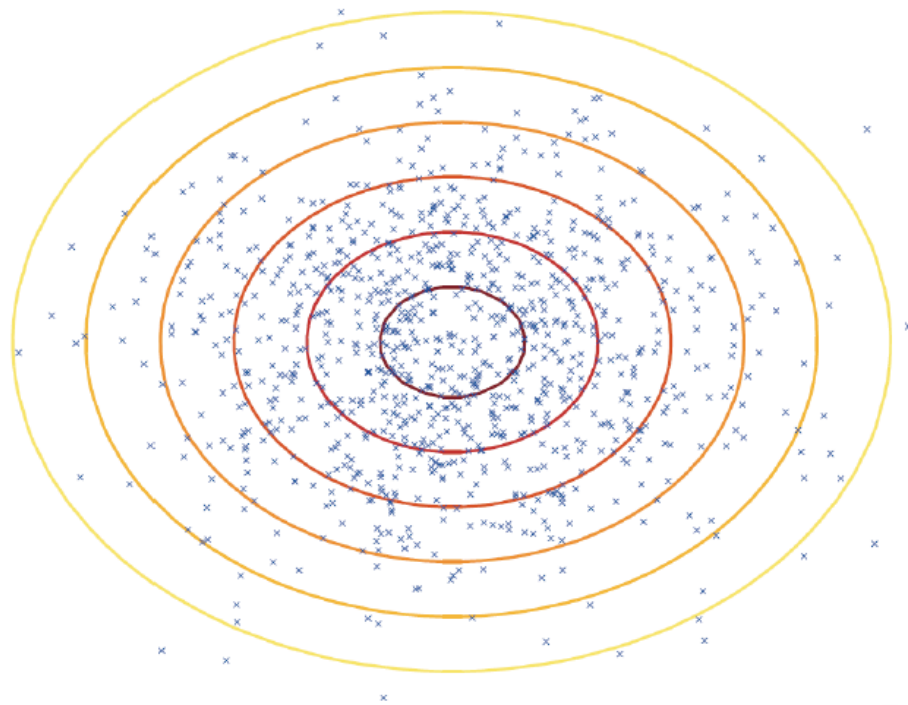
Current project with people in astrophysics





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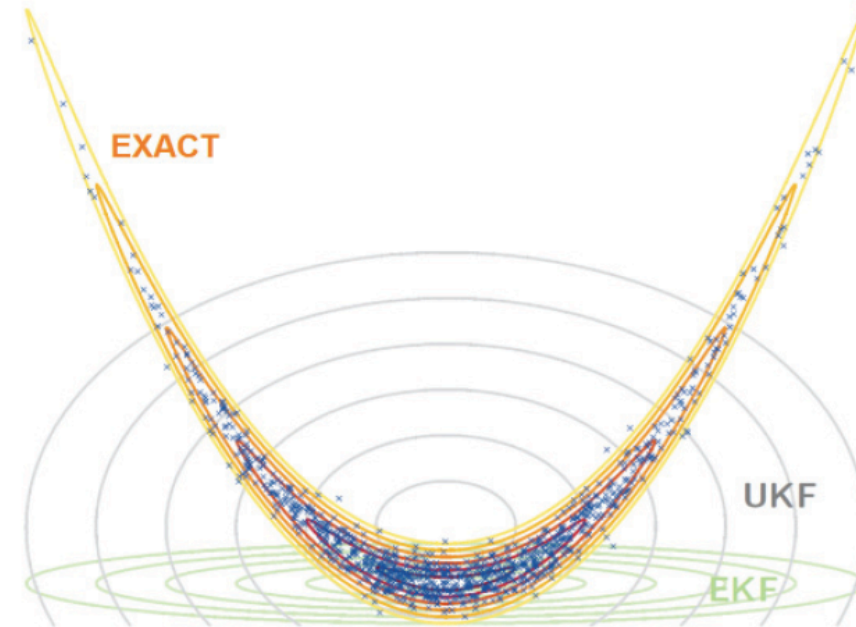
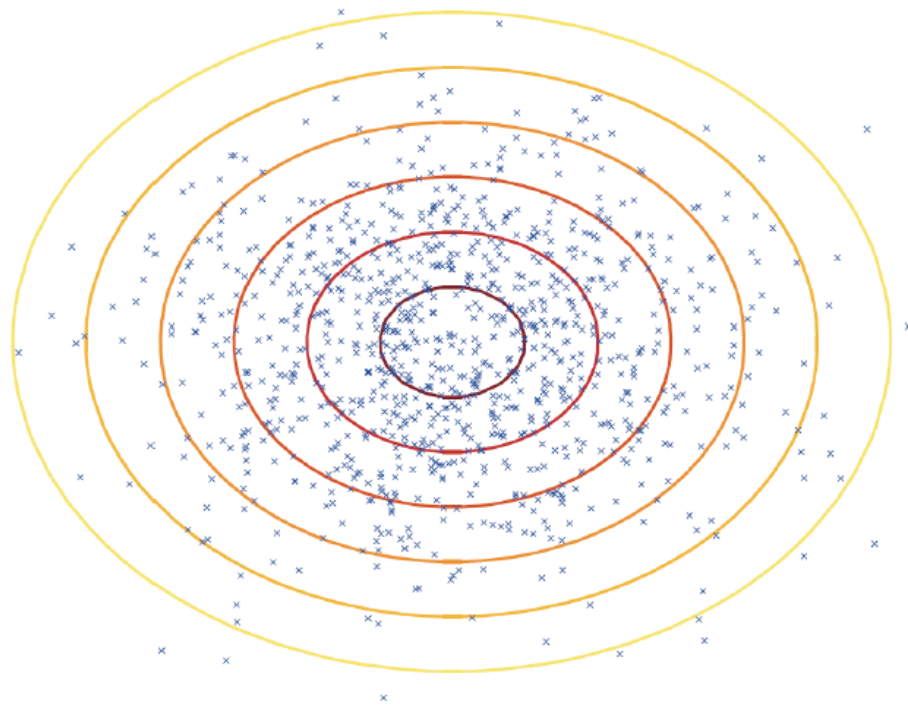
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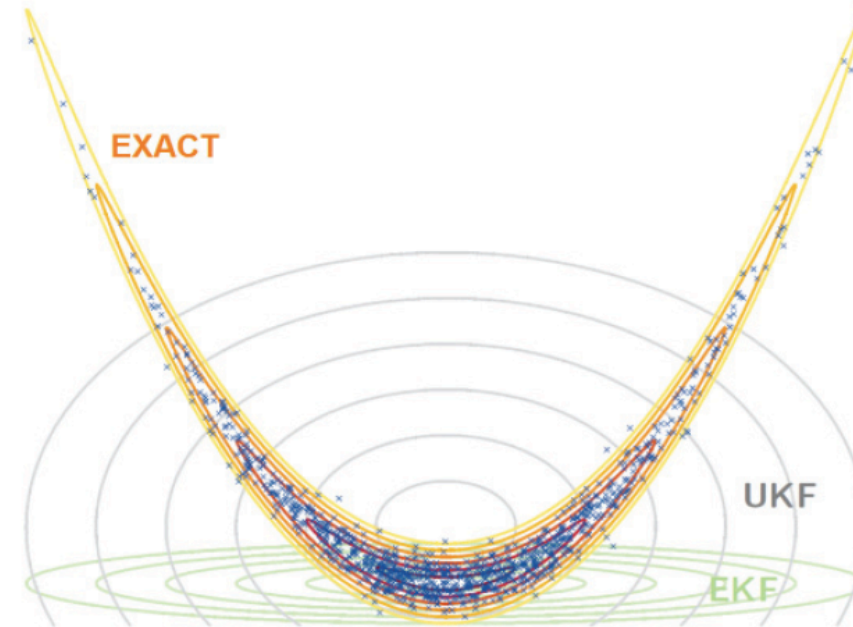
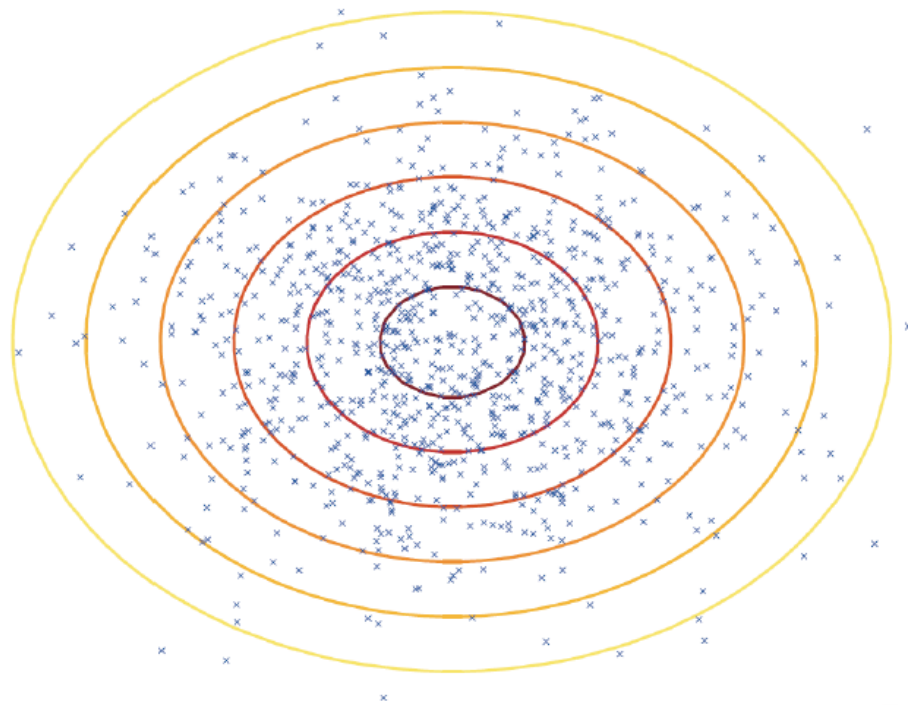
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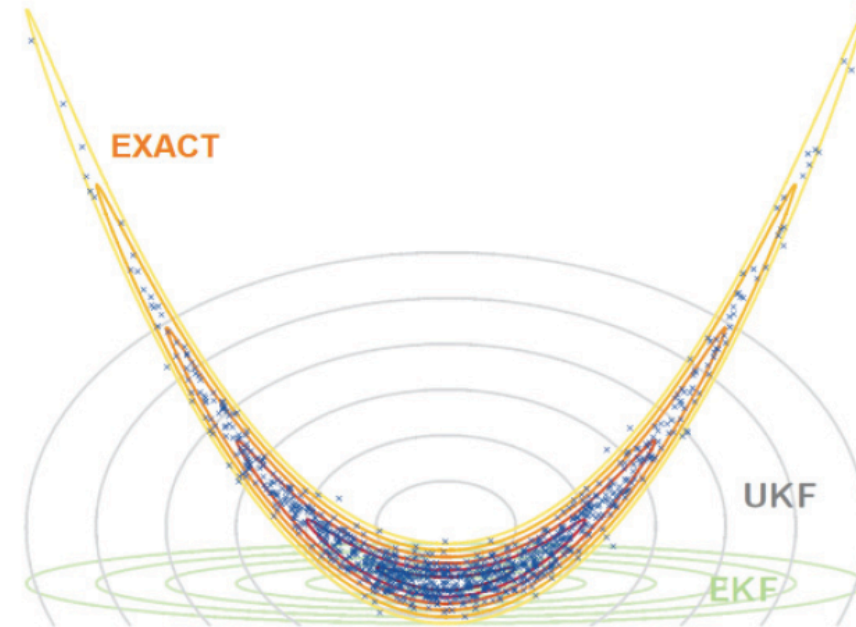
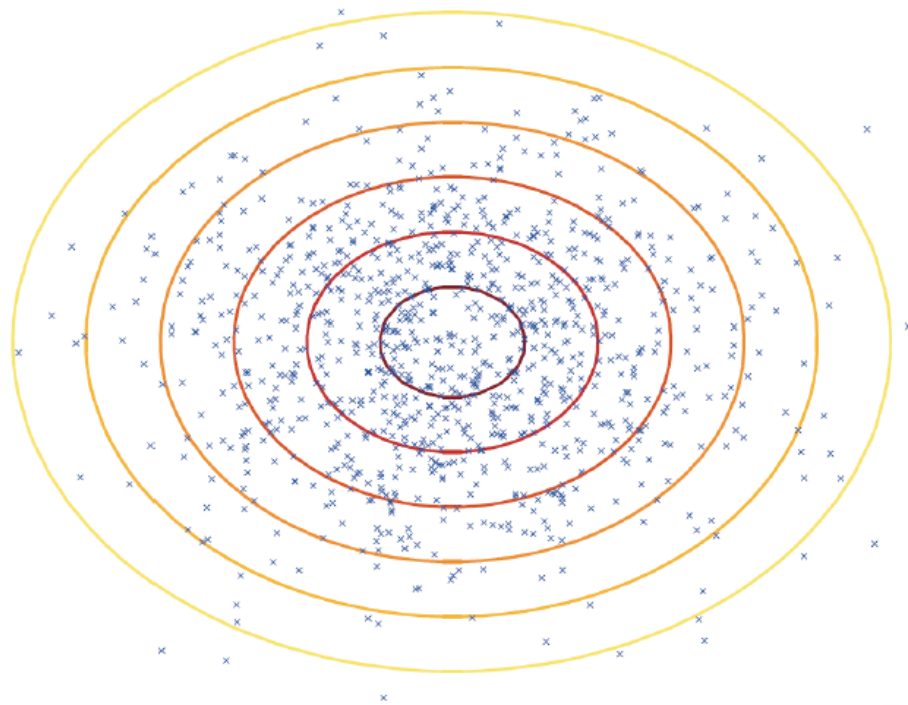
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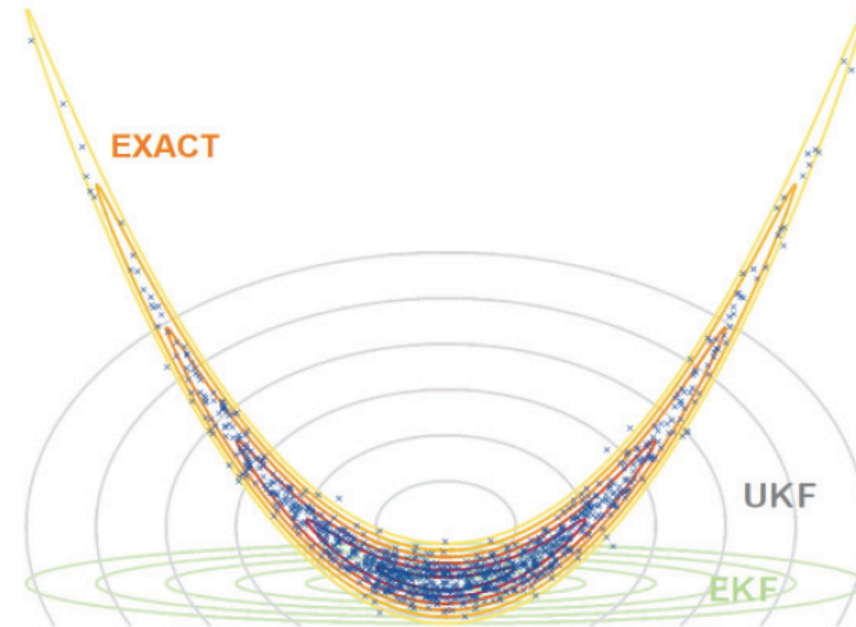
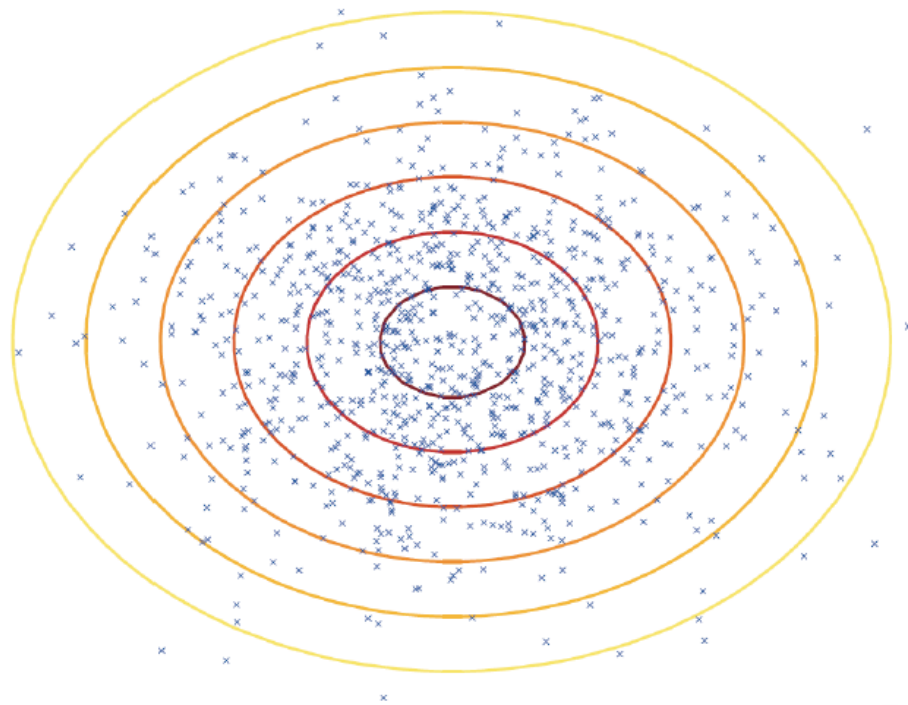
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**Thank you!!**