A particular family of globally periodic birational maps

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In [1] the authors make a classification of a family of birational surface maps. They have identified the subfamilies with dynamical degree $1 \leq D \leq 2$, where

$$D = \lim_{n\to\infty} (\deg(f^n))^{\frac{1}{n}},$$

see [3]. We investigate one of the subfamilies of [1] with $D = 1$. For complex numbers α, β and γ, we consider the following family of birational maps:

$$f(x, y) = \left(\alpha x, \frac{\beta + x}{\gamma + y}\right).$$

Let $F : \mathbf{P}^2 \to \mathbf{P}^2$ be the extension of f in projective space. We define the indeterminacy locus of F as $\mathcal{I} = \{O_0, O_1, O_2\}$. In order to regularize F we blow up all the orbits of points under F that reach any indeterminacy point of F. Trivially there are two orbits of ponits A_0 and A_1 that reach O_2 and O_1 respectively. We then impose the condition on the orbit of A_2 such that $F^p(A_2) = O_0$, where A_0, A_1, A_2 are indeterminacy points of F^{-1}. Let X be the space we get after blowing up operation and let $\tilde{F}$ be the induced map. We find that $\tilde{F}$ is an algebraically stable map which induces a morphism of groups $F^* : Pic(X) \to Pic(X)$. This map F^* is a homeomorphism on $Pic(X)$. We claim that F is a globally periodic map with period $2p+2$ $\forall p \in \mathbf{N}$ provided that $F^p(A_2) = O_0$. We prove this by first showing that $(F^*)^{2p+2}$ is the identity map and then we show that F^{2p+2} is a linear map, by using a result from [5]. Finally by using previous results we prove that $(F)^{2p+2}$ is the identity map.

References

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