Relative equilibria in the four-vortex problem with two pairs of equal vorticities

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We consider the set of relative equilibria in the four-vortex problem where two pairs of vortices have equal strength, that is, $\Gamma_1 = \Gamma_2 = 1$ and $\Gamma_3 = \Gamma_4 = m$ where $m \in \mathbb{R} - \{0\}$ is a parameter. Our main result is that for $m > 0$, the convex configurations all contain a line of symmetry, forming a rhombus or an isosceles trapezoid. The rhombus family exists for all m but the isosceles trapezoid case exists only for m positive. In fact, there exist asymmetric convex configurations when $m < 0$. In contrast with the Newtonian 4-body problem, where the main symmetry result stated above is still unproven, the equations in the vortex case are somewhat easier to handle, allowing for a complete classification of all solutions.