Phase portraits on the Poincaré disc of a SIS model

Regilene D. S. Oliveira[1], Alex C. Rezende[2]

[1] *Department of Mathematics, Instituto de Ciências Matemáticas e de Computação, University of São Paulo, São Carlos, Brazil.*
E-mail: regilene@icmc.usp.br

[2] *Department of Mathematics, Instituto de Ciências Matemáticas e de Computação, University of São Paulo, São Carlos, Brazil.*
E-mail: arezende@icmc.usp.br

In the qualitative theory of ordinary differential equations, we can find many papers whose propose is the classification of all the possible topological phase portraits of a given family of differential system. Most of the studies rely on systems with real parameters and the study consists of outlining their phase portraits by finding out some conditions on the parameters. Here, we studied a susceptible-infectious-susceptible (SIS) model given by $\dot{x} = -bxy - mx + cy + mk$, $\dot{y} = bxy - (m+c)y$, where b, c, k, m are real parameters with $b \neq 0$, $m \neq 0$ [1]. Such system describes an infectious disease from which infected people recover with immunity against reinfection. The integrability of such system has already been studied by Nucci and Leach [3] and Llibre and Valls [2]. We found out two different topological classes of phase portraits.

References

[1] F. Brauer. Basic ideas of mathematical epidemiology. In: C. Castillo-Chavez; S. Blower Sally; P. van den Driessche; D. Kirschner; A. A. Yakubu. *Mathematical approaches for emerging and reemerging infectious diseases*. The IMA volumes in mathematics and its applications, **125**, Springer-Verlag, New York, 2002. 31–65.

[2] J. Llibre; C. Valls. *Integrability of a SIS model*. J. Math. Anal. Appl. **344** (2008), 574–578.

[3] M. C. Nucci; P. G. L. Leach. *An integrable SIS model*. J. Math. Anal. Appl. **290** (2004), 506–518.