Periodic orbits in the Rössler prototype-4 system

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O.E. Rössler introduced several systems in the 1970s as prototypes of the the simplest autonomous differential equations having chaos, the simplicity is in the sense of minimal dimension, minimal number of parameters and minimal nonlinearities.

Here we consider the Rössler prototype-4 system

$$\dot{x} = -y - z, \qquad \dot{y} = x, \qquad \dot{z} = \alpha y(1-y) - \beta z, \tag{1}$$

introduced in [1]. See also the book [2]. This differential system exhibits chaotic motion for the parameter values around $\alpha = \beta = 1/2$, having an strange attractor. In [2] it is numerically showed that in the region of the parameter space giving by small positive values of α and β there are periodic orbits of (1), see Figure 3.8 in page 69 of [2].

In this work, by using the first order averaging theory, we prove the existence of a periodic orbit of system (1) for sufficiently small positive values of the real parameters α and β. This confirms the numerical computations performed in earlier works. We also extend the analysis to new different parameter conditions.

References

[1] O.E. Rössler, *Continuous chaos - four prototype equations*, Ann. New York Acad. Sci. **316** (1979), 376–392.

[2] J.C. Sprott, *Elegant chaos. Algebraically simple chaotic flows*, World Scientific Publishing, 2010.