Nilpotent systems with an inverse integrating factor. Center problem and integrability

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We characterize nilpotent systems whose lowest degree quasi-homogeneous term is $(y, \sigma x^n)^T$, $\sigma = \pm 1$, which have an algebraic inverse integrating factor over $\mathbb{C}((x, y))$. In such cases, we show that the systems admit an inverse integrating factor of the form $(h + \cdots)^q$ with $h = 2\sigma x^{n+1} - (n+1)y^2$ and q a rational number.

We prove that, for n even, the systems with formal inverse integrating factor are formally orbital equivalent to $(\dot{x}, \dot{y})^T = (y, x^n)^T$. In the case n odd, we give a formal normal form that characterizes them. As a consequence, we give the link among the existence of formal inverse integrating factor, center problem and integrability of the considered systems.

References

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