Global phase portraits of cubic systems having a center simultaneously at the origin and at infinity

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The center problem is one of the celebrated problems in the qualitative theory of planar differential equations which is closely related to bifurcation problems of limit cycles. Only quadratic centers are completely known. In this poster we present a systematic classification of a 6-parameter family of cubic differential systems

$$\begin{aligned}\dot{x} &= -y + ax^2 + bxy + cy^2 - y\left(x^2 + y^2\right),\\ \dot{y} &= x + ex^2 + fxy + gy^2 + x\left(x^2 + y^2\right),\end{aligned}$$

having simultaneously a center at the origin and at infinity. First of all such family can be classified in a Hamiltonian subfamily $\{a = b + 2g = f = 0\}$ and a reversible one $\{a = c = f = 0\}$ Next for both classes global phase portraits are classified topologically. Finally it is illustrated how the techniques developed in this study can be used to construct polynomial vector fields with described phase portraits.

References

[1] M. Caubergh, J. Llibre and J. Torregrosa, Global Classifcation of a class of Cubic Vector Fields whose canonical regions are period annuli, Internat. J. Bifur. Chaos Appl. Sci. Engrg., 21(7), 1831-1867, 2011.

[2] M. Caubergh, J. Llibre and J. Torregrosa, Global phase portraits of some reversible cubic centers with collinear or infinitely many singularities. To appear in Internat. J. Bifur. Chaos Appl. Sci. Engrg.

[3] M. Caubergh and J. Torregrosa, Global phase portraits of some reversible cubic centers having non-collinear singgularities. Preprint.