On the trigonometric moment problem

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The trigonometric moment problem arises from the study of one-parameter families of centers in polynomial vector fields. It asks for the classification of the trigonometric polynomials Q which are orthogonal to all powers of a trigonometric polynomial P.

We show that this problem has a simple and natural solution under certain conditions on the monodromy group of the Laurent polynomial associated to P. In the case of real trigonometric polynomials, which is the primary motivation of the problem, our conditions are shown to hold for all trigonometric polynomials of degree 15 or less. In the complex case, we show that there are a small number of exceptional monodromy groups up to degree 30 where the conditions fail to hold and show how counter-examples can be constructed in several of these cases.

References

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