On generation of independent quadratic conserved quantities

Alain Albouy

(in collaboration with Hans Lundmark)

Observatoire de Paris, CNRS, Paris, France

In a Hamiltonian system one can produce a conserved quantity from two conserved quantities by using the Poisson bracket. Jacobi considered this remark as the "deepest discovery by Poisson", while other authors, as Bertrand, remarked that nobody ever discovered a new conserved quantity by using this process.

Hans Lundmark observed a more spectacular way of producing new conserved quantities from two given ones. With his advisor Stefan Rauch-Wojciechowski, they considered another class of equations, that they call the Newton systems, where, in a vector space of dimension n, a force depends on the position and defines the second derivative of the position with respect to time. Then two conserved quantities which are quadratic in the velocities produce $n-2$ other ones. The theorem also works on a spherical space. In the Neumann problem on an n-dimensional sphere, starting with the energy and another quadratic conserved quantity, one produces in this way a (known) system of n quadratic independent conserved quantities in involution.

Recently, we found with Lundmark a simple criterion for the functional independence of conserved quantities produced in such a way. We present the result quite simply, using the "projective dynamics" point of view, i.e. the properties of central projection in dynamics discovered by Appell in 1890.