

Lotka-Volterra map Modifications Notations and preliminaries Lower periodic points Relationship between lower and interior periodic points	Lotka-Volterra map Modifications Notations and preliminaries Lower periodic points Relationship between lower and interior periodic points	
Problem to be solved (modified)	Problem to be solved (modified)	
In 1993 A. N. Sharkovskiĭ formulated some problems concerning the properties of the plain map $[x, y] \mapsto [(y - 2)^2, xy]$. One of the questions was about interior periodic points. This map leaves the plane triangle $\Delta = \{ [x, y] : 0 \le x, 0 \le y, x + y \le 4 \}$ invariant. In 2006 Balibrea, García Guirao, Lampart and Llibre studied a conjugate map	In 1993 A. N. Sharkovskiĭ formulated some problems concerning the properties of the plain map $[x, y] \mapsto [(y-2)^2, xy]$. One of the questions was about interior periodic points. This map leaves the plane triangle $\Delta = \{ [x, y] : 0 \le x, 0 \le y, x + y \le 4 \}$ invariant. In 2006 Balibrea, García Guirao, Lampart and Llibre studied a conjugate map	
${\sf F}:\Delta ightarrow\Delta,[x,y]\mapsto [x(4-x-y),xy]$.	${\sf F}:\Delta ightarrow\Delta,[x,y]\mapsto [x(4-x-y),xy]$.	
They found an interior periodic point with period 4	They found an interior periodic point with period 4 and proved that there are no such points with period 2 and 3.	
Peter Maličký Modified Lotka-Volterra maps	Peter Maličký Modified Lotka-Volterra maps	
Lotka-Volterra map Modifications	Lotka-Volterra map Modifications Notations and preliminaries Lower periodic points	
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Lower periodic points Modifications Relationship between lower and interior periodic points	Lotka-volterra map Modifications Lower periodic points Relationship between lower and interior periodic points	
$\begin{array}{l} \begin{array}{l} \begin{array}{l} \begin{array}{l} \begin{array}{l} \begin{array}{l} \begin{array}{l} \begin{array}{l} $	$\begin{array}{l} & \mbox{Lower periodic points} \\ \hline \mbox{Relationship between lower and interior periodic points} \\ \hline \mbox{Problem to be solved (modified)} \\ & \mbox{In 1993 A. N. Sharkovskiĭ formulated some problems concerning} \\ & \mbox{the properties of the plain map } [x, y] \mapsto [(y - 2)^2, xy]. \mbox{ One of the} \\ & \mbox{questions was about interior periodic points. This map leaves the} \\ & \mbox{plane triangle } \Delta = \{ [x, y] : 0 \leq x, 0 \leq y, x + y \leq 4 \} \mbox{ invariant. In} \\ & \mbox{2006 Balibrea, García Guirao, Lampart and Llibre studied a} \\ \end{array}$	

Lotka-Volterra map Modifications Notations and preliminaries Lower periodic points

Relationship between lower and interior periodic points

Problem to be solved (modified)

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 $F: \Delta \rightarrow \Delta, [x, y] \mapsto [x(4 - x - y), xy]$.

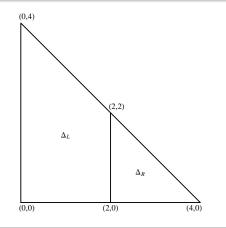
They found an interior periodic point with period 4 and proved that there are no such points with period 2 and 3. In 2012 we published a relation between lower and interior periodic points of the map F. We study the same problem for the modification

 $G: \Delta \rightarrow \Delta, [x, y] \mapsto [x(4 - x - \varphi(x, y)), x\varphi(x, y)]$.

Relationship between lower and interior periodic points

Theorem (Maličký 2012)

Let P be a lower saddle fixed point of the map F^n . Then there is an interior fixed point Q of F^n with the same period and itinerary, where the itinerary is considered with respect to the sets Δ_L and Δ_R .



Notations and preliminaries

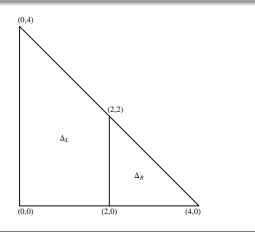
ower periodic points

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Relationship between lower and interior periodic points

Theorem (Maličký 2012)

Let P be a lower saddle fixed point of the map F^n . Then there is an interior fixed point Q of F^n with the same period and itinerary, where the itinerary is considered with respect to the sets Δ_L and Δ_R .



Itinerary

For a fixed point P of the map F^n it is sufficient to consider its itinerary W as a sequence $(w_i)_{i=0}^{n-1}$ defined by

 $w_i = egin{cases} L & ext{if } F^i(P) \in \Delta_L \ R & ext{if } F^i(P) \in \Delta_R \ . \end{cases}$

Such a sequence we will write in a shorten form

Lotka-Volterra map

Modifications

$$W = L^{j_1} R^{k_1} \cdots L^{j_m} R^{k_m}$$

Peter Maličký Modified Lotka-Volterra maps

Lotka-Volterra map Modifications Notations and preliminaries Lower periodic points Relationship between lower and interior periodic points Itinerary	Notation
For a fixed point P of the map F^n it is sufficient to consider its itinerary W as a sequence $(w_i)_{i=0}^{n-1}$ defined by $w_i = \begin{cases} L & \text{if } F^i(P) \in \Delta_L, \\ R & \text{if } F^i(P) \in \Delta_R. \end{cases}$ Such a sequence we will write in a shorten form $W = L^{j_1} R^{k_1} \cdots L^{j_m} R^{k_m}.$	It is natural to express the triangle Δ as the union $\Delta = \Delta_L \cup \Delta_R ,$ where $\Delta_L = \{ [x, y] \in \Delta : x \le 2 \} \text{ and}$ $\Delta_R = \{ [x, y] \in \Delta : x \ge 2 \} ,$
Peter Maličký Modified Lotka-Volterra maps	Notation
It is natural to express the triangle Δ as the union $\Delta = \Delta_L \cup \Delta_R \;,$	It is natural to express the triangle Δ as the union $\Delta = \Delta_L \cup \Delta_R \; ,$
where $\Delta_L = \{ [x, y] \in \Delta : x \le 2 \} \text{ and}$ $\Delta_R = \{ [x, y] \in \Delta : x \ge 2 \},$ because $F(\Delta_L) = \Delta = F(\Delta_R).$	where $\Delta_L = \{ [x, y] \in \Delta : x \le 2 \}$ and $\Delta_R = \{ [x, y] \in \Delta : x \ge 2 \}$, because $F(\Delta_L) = \Delta = F(\Delta_R)$.

nverse maps
The map F is not invertible, but F restricted to $\widetilde{\Delta}_L$ and Δ_R is. The inverse maps of these restrictions are given by $F_L^{-1}: \widetilde{\Delta} \to \widetilde{\Delta}_L, [x, y] \mapsto \left[2 - \sqrt{4 - x - y}, \frac{y}{2 - \sqrt{4 - x - y}}\right]$ $F_R^{-1}: \Delta \to \Delta_R, [x, y] \mapsto \left[2 + \sqrt{4 - x - y}, \frac{y}{2 + \sqrt{4 - x - y}}\right]$
Peter Maličký Modified Lotka-Volterra maps
Lotka-Volterra map Modifications and preliminaries Lower periodic points Relationship between lower and interior periodic point Lower fixed point of <i>F</i> ⁿ
Note that $F : [x, 0] \mapsto [f(x), 0]$, where $f : \langle 0, 4 \rangle \rightarrow \langle 0, 4 \rangle$, $f(x) = x(4 - x)$ is the logistic map, which is conjugate with the tent map $T : \langle 0, 1 \rangle \rightarrow \langle 0, 1 \rangle$, $T(t) = 1 - 1 - 2t $ via the conjugation $h : \langle 0, 1 \rangle \rightarrow \langle 0, 4 \rangle$, $h(t) = 4 \sin^2(\pi t/2)$.

Lotka-Volterra map Modifications Notations and preliminaries Lower periodic points Relationship between lower and interior periodic points Lower fixed point of F ⁿ	Lotka-Volterra map Modifications Notations and preliminaries Lower periodic points Relationship between lower and interior periodic points Lower fixed point of F ⁿ
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Peter Maličký Modified Lotka-Volterra maps Lotka-Volterra map Modifications Notations and preliminaries Lower periodic points Relationship between lower and interior periodic points	Peter Maličký Modified Lotka-Volterra maps Lotka-Volterra map Notations and preliminaries Modifications Nover periodic points Relationship between lower and interior periodic points
Lower fixed point of <i>Fⁿ</i>	Lower fixed point of <i>Fⁿ</i>
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Lotka-Volterra map ModificationsNotations and preliminaries Lower periodic points Relationship between lower and interior periodic points	Lotka-Volterra map Modifications Modifications and preliminaries Lower periodic points Relationship between lower and interior periodic points
Jacobi matrix	Jacobi matrix
Let $P = [x_0, 0] \in \Delta$ be a fixed point of the map F^n . In this case $P = \left[4\sin^2\frac{k\pi}{2^n\pm 1}, 0\right]$. Then the Jacobi matrix of the map F^n at the point P has a form $\left(\begin{array}{c} \lambda_1 & \mu \end{array}\right) = \left(\begin{array}{c} \mp 2^n & \mu \\ \mu & \mu \end{array}\right)$	Let $P = [x_0, 0] \in \Delta$ be a fixed point of the map F^n . In this case $P = \left[4 \sin^2 \frac{k\pi}{2^n \pm 1}, 0\right]$. Then the Jacobi matrix of the map F^n at the point P has a form $\left(\begin{array}{c} \lambda_1 & \mu \end{array}\right) = \left(\begin{array}{c} \mp 2^n & \mu \\ \mu & \mu \end{array}\right)$
$\begin{pmatrix} \lambda_1 & \mu \\ 0 & \lambda_2 \end{pmatrix} = \begin{pmatrix} \mp 2^n & \mu \\ 0 & \prod_{i=0}^{n-1} x_i \end{pmatrix},$ where $[x_i, 0] = F^i(P).$	$\begin{pmatrix} \lambda_1 & \mu \\ 0 & \lambda_2 \end{pmatrix} = \begin{pmatrix} \mp 2^n & \mu \\ 0 & \prod_{i=0}^{n-1} x_i \end{pmatrix},$ where $[x_i, 0] = F^i(P).$
Peter Maličký Modified Lotka-Volterra maps Lotka-Volterra map Modifications Notations and preliminaries Lower periodic points Relationship between lower and interior periodic points Formula for λ ₂	Peter Maličký Modified Lotka-Volterra maps Lotka-Volterra map Notations and preliminaries Lotka-Volterra map Notations and preliminaries Lower periodic points Relationship between lower and interior periodic points Formula for λ2 Value
Since $x_i=4\sin^2rac{2^ik\pi}{2^n\pm 1},$ we have $\lambda_2=\prod_{i=0}^{n-1}4\sin^2rac{2^ik\pi}{2^n\pm 1}.$	Since $x_i=4\sin^2rac{2^ik\pi}{2^n\pm 1}~,$ we have $\lambda_2=\prod_{i=0}^{n-1}4\sin^2rac{2^ik\pi}{2^n\pm 1}~.$
Peter Maličký Modified Lotka-Volterra maps	Peter Maličký Modified Lotka-Volterra maps

Lotka-Volterra map Modifications	Notations and preliminaries Lower periodic points Relationship between lower and interior periodic points	Lotka-Volterra map Modifications	Notations and preliminaries Lower periodic points Relationship between lower and interior periodic points
Classification		Classification	
For λ_2 we have the possibilities		For λ_2 we have the possibilities	
Saddle point		Saddle point	
		$0\leq\lambda_2<1$,	
Peter Maličký	Modified Lotka-Volterra maps	Peter Maličký	Modified Lotka-Volterra maps
Lotka-Volterra map Modifications	Notations and preliminaries Lower periodic points Relationship between lower and interior periodic points	Lotka-Volterra map Modifications	Notations and preliminaries Lower periodic points Relationship between lower and interior periodic points
Classification		Classification	
For λ_2 we have the possibilities		For λ_2 we have the possibilities	
Saddle point		Saddle point	
$0 \le \lambda_2 < 1$, e.g. $x_0 = 4 \sin^2 \frac{\pi}{17}$		$0 \le \lambda_2 < 1$, e.g. $x_0 = 4 \sin^2 \frac{\pi}{17}$	
		Nonhyperbolic point	
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Lotka-Volterra map Modifications	Notations and preliminaries Lower periodic points Relationship between lower and interior periodic points	Lotka-Volterra map Modifications	Notations and preliminaries Lower periodic points Relationship between lower and interior periodic points
For λ_2 we have the possibilities		For λ_2 we have the possibilities	
Saddle point		Saddle point	
$0 \le \lambda_2 < 1$, e.g. $x_0 = 4 \sin^2 \frac{\pi}{17}$		$0 \le \lambda_2 < 1$, e.g. $x_0 = 4 \sin^2 \frac{\pi}{17}$	
Nonhyperbolic point		Nonhyperbolic point	
$\lambda_2 = 1$, e.g. $x_0 = 4\sin^2\frac{\pi}{15}$		$\lambda_2 = 1$, e.g. $x_0 = 4 \sin^2 \frac{\pi}{15}$	
Repulsive point		Repulsive point	
$1 < \lambda_2$, e.g. $x_0 = 4 \sin^2 rac{3\pi}{17}$		$1 < \lambda_2$, e.g. $x_0 = 4 \sin^2 \frac{3\pi}{17}$	
		Remark All above points $[x_0, 0]$ have period	od 4.
Peter Maličký	Modified Lotka-Volterra maps	Peter Maličký	Modified Lotka-Volterra maps
Lotka-Volterra map Modifications	Notations and preliminaries Lower periodic points Relationship between lower and interior periodic points	Lotka-Volterra map Modifications	Notations and preliminaries Lower periodic points Relationship between lower and interior periodic points
For λ_2 we have the possibilities			
Saddle point		Saddle point	
$0 \le \lambda_2 < 1$, e.g. $x_0 = 4 \sin^2 \frac{\pi}{17}$		Lower periodic points with period $n \ge 4$.	n and $0 < \lambda_2 < 1$ appear for all
Nonhyperbolic point			
$\lambda_2 = 1$, e.g. $x_0 = 4\sin^2 \frac{\pi}{15}$			
Repulsive point			
$1 < \lambda_2$, e.g. $x_0 = 4 \sin^2 \frac{3\pi}{17}$			
Remark			
All above points $[x_0, 0]$ have period	1 4.		
Peter Maličký	Modified Lotka-Volterra maps	Peter Maličký	Modified Lotka-Volterra maps

Lotka-Volterra map Modifications Relationship between lower and interior periodic points	Lotka-Volterra map Modifications Relationship between lower and interior periodic points
Classification	Classification
Saddle point	Saddle point
Lower periodic points with period <i>n</i> and $0 < \lambda_2 < 1$ appear for all $n \ge 4$.	Lower periodic points with period <i>n</i> and $0 < \lambda_2 < 1$ appear for all $n \ge 4$.
Nonhyperbolic point	Nonhyperbolic point
	Lower periodic points with period <i>n</i> and $\lambda_2 = 1$ appear for
	infinitely many n , e.g. $n = 4 \cdot 3^i \cdot 5^j$, where $i \ge 0$, $j \ge 0$.
Peter Maličký Modified Lotka-Volterra maps	Peter Maličký Modified Lotka-Volterra maps
Lotka-Volterra map Modifications Relationship between lower and interior periodic points	Lotka-Volterra map Modifications Modifications Relationship between lower and interior periodic points
Classification	Classification
Saddle point $1 = 1 = 1 = 1$	Saddle point $(1 - 1) = 0$
Lower periodic points with period <i>n</i> and $0 < \lambda_2 < 1$ appear for all $n \ge 4$.	Lower periodic points with period n and $0 < \lambda_2 < 1$ appear for all $n \ge 4$.
Nonhyperbolic point	Nonhyperbolic point
	Lower periodic points with period <i>n</i> and $\lambda_2 = 1$ appear for
Lower periodic points with period <i>n</i> and $\lambda_2 = 1$ appear for	
Lower periodic points with period n and $\lambda_2 = 1$ appear for infinitely many n , e.g. $n = 4 \cdot 3^i \cdot 5^j$, where $i \ge 0$, $j \ge 0$.	infinitely many <i>n</i> , e.g. $n = 4 \cdot 3^i \cdot 5^j$, where $i \ge 0, j \ge 0$.
	infinitely many <i>n</i> , e.g. $n = 4 \cdot 3^i \cdot 5^j$, where $i \ge 0$, $j \ge 0$. Repulsive point
infinitely many <i>n</i> , e.g. $n = 4 \cdot 3^i \cdot 5^j$, where $i \ge 0, j \ge 0$.	infinitely many n , e.g. $n = 4 \cdot 3^i \cdot 5^j$, where $i \ge 0, j \ge 0$.

Peter Maličký Modified Lotka-Volterra maps

Modified Lotka-Volterra maps

Peter Maličký

Lotka-Volterra map Modifications Relationship between lower and interior periodic points	Lotka-Volterra map Modifications Relationship between lower and interior periodic points
Classification	Main result
Saddle point Lower periodic points with period n and $0 < \lambda_2 < 1$ appear for all $n \ge 4$. Nonhyperbolic point Lower periodic points with period n and $\lambda_2 = 1$ appear for infinitely many n , e.g. $n = 4 \cdot 3^i \cdot 5^j$, where $i \ge 0$, $j \ge 0$. Repulsive point Lower periodic points with period n and $1 < \lambda_2$ appear for all $n \ge 1$.	Theorem (Maličký 2012) Let P be a lower saddle fixed point of the map F^n . Then there is an interior fixed point Q of F^n with the same period and itinerary. The proof is based on the Brouwer fixed point theorem.
Peter Maličký Modified Lotka-Volterra maps Lotka-Volterra map Modifications Notations and preliminaries Lower periodic points Relationship between lower and interior periodic points Main result	Peter Maličký Modified Lotka-Volterra maps Lotka-Volterra map Modifications Notations and preliminaries Lower periodic points Relationship between lower and interior periodic points Main result Main result
Theorem (Maličký 2012) Let P be a lower saddle fixed point of the map F^n . Then there is an interior fixed point Q of F^n with the same period and itinerary.	Theorem (Maličký 2012) Let P be a lower saddle fixed point of the map F^n . Then there is an interior fixed point Q of F^n with the same period and itinerary.
The proof is based on the Brouwer fixed point theorem. We have also proved that for some itineraries interior fixed points of F^n do not exist.	The proof is based on the Brouwer fixed point theorem. We have also proved that for some itineraries interior fixed points of F^n do not exist.
Peter Maličký Modified Lotka-Volterra maps	Peter Maličký Modified Lotka-Volterra maps

Lotka-Volterra map Modifications	Lotka-Volterra map Modifications
Assume that for any $x \in (0, 4)$ we have an increasing homeomorphism φ_x of the interval $\langle 0, 4 - x \rangle$ onto itself. Moreover let the function $\varphi(x, y) = \varphi_x(y)$ be continuous in the domain $\widehat{\Delta} = \{ [x, y] : 0 < x < 4, 0 \le y \le 4 - x \}$.	Assume that for any $x \in (0, 4)$ we have an increasing homeomorphism φ_x of the interval $\langle 0, 4 - x \rangle$ onto itself. Moreover let the function $\varphi(x, y) = \varphi_x(y)$ be continuous in the domain $\widehat{\Delta} = \{ [x, y] : 0 < x < 4, 0 \le y \le 4 - x \}.$
Peter Maličký Modified Lotka-Volterra maps Lotka-Volterra map Modifications Image: Modification state Modifications Image: Modification state	Peter Maličký Modified Lotka-Volterra maps Lotka-Volterra map Modifications Modifications
To obtain such above family φ_x , choose for $0 < x < 4$ a family of increasing homeomorphisms ψ_x of the interval $\langle 0, 1 \rangle$ such that the function $\psi(x, y) = \psi_x(y)$ is continuous in $(0, 4) \times \langle 0, 1 \rangle$ and put $\varphi_x(y) = (4 - x)\psi_x\left(\frac{y}{4 - x}\right)$.	To obtain such above family φ_x , choose for $0 < x < 4$ a family of increasing homeomorphisms ψ_x of the interval $\langle 0, 1 \rangle$ such that the function $\psi(x, y) = \psi_x(y)$ is continuous in $(0, 4) \times \langle 0, 1 \rangle$ and put $\varphi_x(y) = (4 - x)\psi_x\left(\frac{y}{4 - x}\right)$.

Lotka-Volterra map Modifications Modifications	Lotka-Volterra map Modifications Modifications
It is natural to put $\varphi_4(0)=0$. On the other hand we assume nothing about existence and properties of the limit $\lim_{x\to 0} \varphi_x(y).$	It is natural to put $\varphi_4(0)=0$. On the other hand we assume nothing about existence and properties of the limit $\lim_{x\to 0} \varphi_x(y).$
Peter Maličký Modified Lotka-Volterra maps Lotka-Volterra map Modifications Image: Constant of the second	Peter Maličký Modified Lotka-Volterra maps Lotka-Volterra map Modifications Modifications
Let $G : \Delta \to \Delta$ be defined by $G[x, y] = \begin{cases} [0, 0] & \text{if } x = 0, \\ [x(4 - x - \varphi_x(y)), x\varphi_x(y)] & \text{otherwise} \end{cases}.$ Then G is called a modified Lotka–Volterra map.	Let $G : \Delta \to \Delta$ be defined by $G[x, y] = \begin{cases} [0, 0] & \text{if } x = 0, \\ [x(4 - x - \varphi_x(y)), x\varphi_x(y)] & \text{otherwise} \end{cases}.$ Then G is called a modified Lotka–Volterra map.

Modifications Modifications All such modifications have properties • G is continuous on Δ • G (Δ_L) = Δ = G(Δ_R) • Working • Control (Δ_R) • Modifications • G (Δ_L) = Δ = G(Δ_R) • Control (Δ_R) • Control (Δ_R) • Control (Δ_R) • Control (Δ_R) • Control (Δ_R) • Control (Δ_R) • Control (Δ_R) • Control (Δ_R) • Control (Δ_R) • Control (Δ_R) • Control (Δ_R) • Control (Δ_R) • Continuous on Δ • Continuous on	Lotka-Volterra map Modifications	Lotka-Volterra map Modifications
• <i>G</i> is continuous on Δ • <i>G</i> is continuous on Δ • <i>G</i> (Δ_L) = Δ = <i>G</i> (Δ_R) • <i>G</i> (Δ_L) = Δ = <i>G</i> (Δ_R) • <i>G</i> (Δ_L) = Δ = <i>G</i> (Δ_R) • <i>G</i> (Δ_L) = Δ = <i>G</i> (Δ_R) • <i>G</i> (Δ_L) = Δ = <i>G</i> (Δ_R) • <i>G</i> (Δ_L) = Δ = <i>G</i> (Δ_R) • <i>G</i> (Δ_L) = Δ = <i>G</i> (Δ_R) • <i>G</i> (Δ_L) = Δ = <i>G</i> (Δ_R) • <i>G</i> restricted to $\overline{\Delta}_L$ and Δ_R is invertible • <i>G</i> is continuous on Δ • <i>G</i> (Δ_L) = Δ = <i>G</i> (Δ_R) • <i>G</i> restricted to $\overline{\Delta}_L$ and Δ_R is invertible • <i>G</i> is continuous on Δ • <i>G</i> (Δ_L) = Δ = <i>G</i> (Δ_R) • <i>G</i> restricted to $\overline{\Delta}_L$ and Δ_R is invertible • The inverse maps of these restrictions are given by $G_L^{-1}: \overline{\Delta} \to \overline{\Delta}_L,$ $[x, y] + [2 - \sqrt{4 - x - y}, \varphi_{Z-\sqrt{4 - x - y}}^{-1}(\frac{y}{2-\sqrt{4 - x - y}})]$	Modifications	Modifications
Lotte-Volterra map Modifications Modifications Lotte-Volterra map Modifications Modifications Modifications All such modifications have properties All such modifications have properties • G is continuous on Δ • G (Δ_L) = Δ = $G(\Delta_R)$ • G is continuous on Δ • G (Δ_L) = Δ = $G(\Delta_R)$ • G restricted to $\widetilde{\Delta}_L$ and Δ_R is invertible • G restricted to $\widetilde{\Delta}_L$ and Δ_R is invertible • The inverse maps of these restrictions are given by $G_L^{-1}: \widetilde{\Delta} \to \widetilde{\Delta}_L$, $[x, y] \mapsto \left[2 - \sqrt{4 - x - y}, \varphi_{2-\sqrt{4 - x - y}}^{-1} \left(\frac{y}{2 - \sqrt{4 - x - y}}\right)\right]$		• G is continuous on Δ
ModificationsModificationsModificationsModificationsAll such modifications have propertiesAll such modifications have properties• G is continuous on Δ G is continuous on Δ • $G(\Delta_L) = \Delta = G(\Delta_R)$ $G(\Delta_L) = \Delta = G(\Delta_R)$ • G restricted to $\widetilde{\Delta}_L$ and Δ_R is invertible• G restricted to $\widetilde{\Delta}_L$ and Δ_R is invertible• The inverse maps of these restrictions are given by $G_L^{-1}: \widetilde{\Delta} \to \widetilde{\Delta}_L,$ $[x, y] \mapsto \left[2 - \sqrt{4 - x - y}, \varphi_{2-\sqrt{4 - x - y}}^{-1} \left(\frac{y}{2-\sqrt{4 - x - y}}\right)\right]$		
All such modifications have properties • <i>G</i> is continuous on Δ • $G(\Delta_L) = \Delta = G(\Delta_R)$ • <i>G</i> restricted to $\widetilde{\Delta}_L$ and Δ_R is invertible • <i>G</i> restricted to $\widetilde{\Delta}_L$ and Δ_R is invertible • <i>G</i> restricted to $\widetilde{\Delta}_L$ and Δ_R is invertible • The inverse maps of these restrictions are given by $G_L^{-1}: \widetilde{\Delta} \to \widetilde{\Delta}_L,$ $[x, y] \mapsto \left[2 - \sqrt{4 - x - y}, \varphi_{2-\sqrt{4 - x - y}}^{-1} \left(\frac{y}{2-\sqrt{4 - x - y}}\right)\right]$	Modifications	Modifications
	All such modifications have properties • G is continuous on Δ • $G(\Delta_L) = \Delta = G(\Delta_R)$	All such modifications have properties • <i>G</i> is continuous on Δ • $G(\Delta_L) = \Delta = G(\Delta_R)$ • <i>G</i> restricted to $\widetilde{\Delta}_L$ and Δ_R is invertible • The inverse maps of these restrictions are given by $G_L^{-1}: \widetilde{\Delta} \to \widetilde{\Delta}_L,$ $[x, y] \mapsto \left[2 - \sqrt{4 - x - y}, \varphi_{2-\sqrt{4 - x - y}}^{-1}\left(\frac{y}{2-\sqrt{4 - x - y}}\right)\right]$

Modifications

All such modifications have properties

- G is continuous on Δ
- $G(\Delta_I) = \Delta = G(\Delta_R)$
- G restricted to $\widetilde{\Delta}_{I}$ and Δ_{R} is invertible

Lotka-Volterra map

Modifications

• The inverse maps of these restrictions are given by

$$\begin{split} G_L^{-1} &: \widetilde{\Delta} \to \widetilde{\Delta}_L \,, \\ [x, y] &\mapsto \left[2 - \sqrt{4 - x - y}, \, \varphi_{2-\sqrt{4-x-y}}^{-1} \left(\frac{y}{2-\sqrt{4-x-y}} \right) \right] \\ G_R^{-1} &: \Delta \to \Delta_R \,, \\ [x, y] &\mapsto \left[2 + \sqrt{4 - x - y}, \, \varphi_{2+\sqrt{4-x-y}}^{-1} \left(\frac{y}{2+\sqrt{4-x-y}} \right) \right] \end{split}$$

Modified Lotka-Volterra maps

• G restricted to the lower side is a logistic map.

Peter Maličký

Modifications

Definition

All such modifications have properties

- G is continuous on Δ
- $G(\Delta_I) = \Delta = G(\Delta_R)$
- G restricted to $\widetilde{\Delta}_{I}$ and Δ_{R} is invertible
- The inverse maps of these restrictions are given by

$$\begin{split} & G_L^{-1}: \widetilde{\Delta} \to \widetilde{\Delta}_L \,, \\ & [x, \, y] \mapsto \left[2 - \sqrt{4 - x - y}, \, \varphi_{2-\sqrt{4-x-y}}^{-1} \left(\frac{y}{2-\sqrt{4-x-y}} \right) \right] \\ & G_R^{-1}: \Delta \to \Delta_R, \\ & [x, \, y] \mapsto \left[2 + \sqrt{4 - x - y}, \, \varphi_{2+\sqrt{4-x-y}}^{-1} \left(\frac{y}{2+\sqrt{4-x-y}} \right) \right] \end{split}$$

Modified Lotka-Volterra maps

• *G* restricted to the lower side is a logistic map. Peter Maličký

Let $G^n[x, y] = [g_n(x, y), h_n(x, y)]$ and $P = [x_0, 0]$ be a lower fixed

 $h_n(x, y) > y$ (respectively $h_n(x, y) < y$)

 $h_n(x, y) > ky$ (respectively $h_n(x, y) < ky$)

with k > 1 (respectively 0 < k < 1) than *P* is called strictly

Lotka-Volterra map Modifications

point of the map G^n . The point P is called a repulsive

(respectively saddle) point if there is $\delta > 0$ such that

Repulsive and saddle points fixed points

for all $[x, y] \in (x_0 - \delta, x_0 + \delta) \times (0, \delta)$.

repulsive (respectively strict saddle) point.

Lotka-Volterra map Modifications

Repulsive and saddle points fixed points

Definition

Let $G^n[x, y] = [g_n(x, y), h_n(x, y)]$ and $P = [x_0, 0]$ be a lower fixed point of the map G^n . The point P is called a repulsive (respectively saddle) point if there is $\delta > 0$ such that

 $h_n(x, y) > y$ (respectively $h_n(x, y) < y$)

for all $[x, y] \in (x_0 - \delta, x_0 + \delta) \times (0, \delta)$.

Lotka-Volterra map Modifications epulsive and saddle points fixed points	Lotka-Volterra map Modifications Main result for modified Lotka–Volterra maps
Definition Let $G^n[x, y] = [g_n(x, y), h_n(x, y)]$ and $P = [x_0, 0]$ be a lower fixed point of the map G^n . The point P is called a repulsive (respectively saddle) point if there is $\delta > 0$ such that $h_n(x, y) > y$ (respectively $h_n(x, y) < y$) for all $[x, y] \in (x_0 - \delta, x_0 + \delta) \times (0, \delta)$. If the above inequalities can be replaced by $h_n(x, y) > ky$ (respectively $h_n(x, y) < ky$) with $k > 1$ (respectively $0 < k < 1$) than P is called strictly repulsive (respectively strict saddle) point.	Theorem Let $P \neq [0,0]$ be a lower saddle fixed point of the map G^n . Then there is an interior fixed point Q of G^n with the same period and itinerary.
Peter Maličký Modified Lotka-Volterra maps	Peter Maličký Modified Lotka-Volterra maps
ormula for λ_2	Formula for λ_2
We have $\widetilde{\lambda}_{2} = \prod_{i=0}^{n-1} x_{i} \varphi_{x_{i}}'(0) = \prod_{i=0}^{n-1} x_{i} \psi_{x_{i}}'(0) ,$ or equivalently $\widetilde{\lambda}_{2} = \prod_{i=0}^{n-1} x_{i} \frac{\partial \varphi}{\partial y}(x_{i}, 0) = \prod_{i=0}^{n-1} x_{i} \frac{\partial \psi}{\partial y}(x_{i}, 0) ,$	We have $\widetilde{\lambda}_{2} = \prod_{i=0}^{n-1} x_{i} \varphi'_{x_{i}}(0) = \prod_{i=0}^{n-1} x_{i} \psi'_{x_{i}}(0) ,$ or equivalently $\widetilde{\lambda}_{2} = \prod_{i=0}^{n-1} x_{i} \frac{\partial \varphi}{\partial y}(x_{i}, 0) = \prod_{i=0}^{n-1} x_{i} \frac{\partial \psi}{\partial y}(x_{i}, 0) ,$
where $\varphi(x,y)=\varphi_x(y)$ and	where $\varphi(x, y) = \varphi_x(y)$ and $\psi(x, y) = \psi_x(y)$.

Lotka-Volterra map	
Formula for λ_2	
We have $\widetilde{\lambda}_{2} = \prod_{i=0}^{n-1} x_{i} \varphi'_{x_{i}}(0) = \prod_{i=0}^{n-1} x_{i} \psi'_{x_{i}}(0) ,$ or equivalently $\widetilde{\lambda}_{2} = \prod_{i=0}^{n-1} x_{i} \frac{\partial \varphi}{\partial y}(x_{i}, 0) = \prod_{i=0}^{n-1} x_{i} \frac{\partial \psi}{\partial y}(x_{i}, 0) ,$ where $\varphi(x, y) = \varphi_{x}(y)$ and $\psi(x, y) = \psi_{x}(y)$.	(i) Let $0 \le a \le 2$ and $\psi_x(y) = ay + (1-a)y^2$. Then we obtain $\varphi_x(y) = ay + \frac{(1-a)y^2}{4-x}$. (ii) Let $\psi_x(y) = \frac{\sqrt{2y+x^2}-\sqrt{y+x^2}}{\sqrt{2+x^2}-\sqrt{1+x^2}}$. Then we obtain $\varphi_x(y) = \frac{\sqrt{2y(4-x) + x^2(4-x)^2} - \sqrt{y(4-x) + x^2(4-x)^2}}{\sqrt{2+x^2} - \sqrt{1+x^2}}$.
Peter Maličký Modified Lotka-Volterra maps	
(i) Let $0 \le a \le 2$ and $\psi_x(y) = ay + (1-a)y^2$. Then we obtain $\varphi_x(y) = ay + \frac{(1-a)y^2}{4-x}$. (ii) Let $\psi_x(y) = \frac{\sqrt{2y+x^2} - \sqrt{y+x^2}}{\sqrt{2+x^2} - \sqrt{1+x^2}}$. Then we obtain $\varphi_x(y) = \frac{\sqrt{2y(4-x) + x^2(4-x)^2} - \sqrt{y(4-x) + x^2(4-x)^2}}{\sqrt{2+x^2} - \sqrt{1+x^2}}$.	(i) Let $0 \le a \le 2$ and $\psi_x(y) = ay + (1-a)y^2$. Then we obtain $\varphi_x(y) = ay + \frac{(1-a)y^2}{4-x}$. (ii) Let $\psi_x(y) = \frac{\sqrt{2y+x^2}-\sqrt{y+x^2}}{\sqrt{2+x^2}-\sqrt{1+x^2}}$. Then we obtain $\varphi_x(y) = \frac{\sqrt{2y(4-x)+x^2(4-x)^2}-\sqrt{y(4-x)+x^2(4-x)^2}}{\sqrt{2+x^2}-\sqrt{1+x^2}}$.
(iii) Let $\psi_x(y) = \sqrt{y}$. Then $\varphi_x(y) = \sqrt{(4-x)y}$.	(iii) Let $\psi_x(y) = \sqrt{y}$. Then $\varphi_x(y) = \sqrt{(4-x)y}$. (iv) Let $\psi_x(y) = y^x$. Then $\varphi_x(y) = (4-x) \left(\frac{y}{4-x}\right)^x$.

(i) Let
$$0 \le a \le 2$$
 and $\varphi_{i}(y) = ay + (1-a)y^{2}$. Then we obtain
 $\varphi_{i}(y) = ay + (\frac{1-a}{2})x^{2}$.
(ii) Let $\psi_{i}(y) = \sqrt{y^{2}(4-x) + x^{2}(4-x)^{2}} - \sqrt{y(4-x) + x^{2}(4-x)^{2}}$.
(iii) Let $\psi_{i}(y) = \sqrt{y}$. Then $\varphi_{i}(y) = \sqrt{y(4-x) + x^{2}(4-x)^{2}}}$.
(iii) Let $\psi_{i}(y) = \sqrt{y}$. Then $\varphi_{i}(y) = \sqrt{(4-x)}y$.
(iv) Let $\psi_{i}(y) = y^{x}$. Then $\varphi_{i}(y) = (4-x)\left(\frac{x}{x^{2}}\right)^{x}$.
(iii) Let $\psi_{i}(y) = y^{x}$. Then $\varphi_{i}(y) = (4-x)\left(\frac{x}{x^{2}}\right)^{x}$.
(iii) Let $\psi_{i}(y) = y^{x}$. Then $\varphi_{i}(y) = (4-x)\left(\frac{x}{x^{2}}\right)^{x}$.
(iii) Let $\psi_{i}(y) = y^{x}$. Then $\varphi_{i}(y) = (4-x)\left(\frac{x}{x^{2}}\right)^{x}$.
(iii) Let $\psi_{i}(y) = y^{x}$. Then $\varphi_{i}(y) = (4-x)\left(\frac{x}{x^{2}}\right)^{x}$.
(iii) Let $\phi_{i}(y) = y^{x}$. Then $\varphi_{i}(y) = (4-x)\left(\frac{x}{x^{2}}\right)^{x}$.
(iii) Let $\phi_{i}(y) = y^{x}$. Then $\varphi_{i}(y) = (4-x)\left(\frac{x}{x^{2}}\right)^{x}$.
(if $x = 4$,
 $\int_{x} 2 - a^{x}\lambda_{2}$.
(if $x = 4$,
 $\int_{x} 2 - a^{x}\lambda_{2}$.
(if $0 \le a \le 1/3$ then C^{x} has interior fixed points of C^{x} are saddle
fixed points.
(if $0 \le a < 1/3$ then C^{x} has interior fixed points of C^{x} are saddle
fixed points.
(if $0 \le 1/3$ then C^{x} has interior fixed points of C^{x} are saddle
fixed points.
(if $0 \le 1/3$ then C^{x} has interior fixed points of C^{x} are saddle
fixed points.
(if $0 \le 1/3$ then C^{x} has interior fixed points of C^{x} are saddle
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(if $0 \le 1/3$ then C^{x} has interior fixed points of C^{x} are saddle
fixed points.
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fixed points.
(if $0 \ge 1/3$ then C^{x} has interior fixed points of C^{x} are saddle
fixed points.
(if $0 \ge 1/3$ then C^{x} has interior fixed points of C^{x} are saddle
fixed points.
(if $0 \le 1/3$ then C^{x} has interior fixed point of C and
there is no interior fixed points of C^{x} are saddle
fixed points.
(if $0 \ge 1/3$ then C^{x} has interior fixed point of C and

Lotka-Volterra map Modifications	Lotka-Volterra map Modifications
Modification (i)	Modification (i)
Let $0 \leq a \leq 2$ and $G: \Delta ightarrow \Delta$ be defined by	Let $0 \leq a \leq 2$ and $G: \Delta ightarrow \Delta$ be defined by
$G[x,y] = \begin{cases} [0,0] & \text{if } x = 4, \\ \left[x \left(4 - x - ay - \frac{(1-a)y^2}{4-x} \right), x \left(ay + \frac{(1-a)y^2}{4-x} \right) \right] & \text{otherwise} . \end{cases}$	$G[x,y] = \begin{cases} [0,0] & \text{if } x = 4, \\ \left[x \left(4 - x - ay - \frac{(1-a)y^2}{4-x} \right), x \left(ay + \frac{(1-a)y^2}{4-x} \right) \right] & \text{otherwise} . \end{cases}$
In this case $\widetilde{\lambda}_2 = \textit{a}^n \lambda_2 \; .$	In this case $\widetilde{\lambda}_2 = a^n \lambda_2 \; .$
 If 0 ≤ a < 1/3 then Gⁿ has interior fixed points for all itineraries, because all lower fixed points of Gⁿ are saddle fixed points. If a = 1/3 then [3,0] is not lower saddle fixed point of G and there is no interior fixed point of G lying in Δ_R. The other lower fixed points of Gⁿ are saddle points for any n > 1. 	 If 0 ≤ a < 1/3 then Gⁿ has interior fixed points for all itineraries, because all lower fixed points of Gⁿ are saddle fixed points. If a = 1/3 then [3,0] is not lower saddle fixed point of G and there is no interior fixed point of G lying in Δ_R. The other lower fixed points of Gⁿ are saddle points for any n > 1.
Peter Maličký Modified Lotka-Volterra maps	Peter Maličký Modified Lotka-Volterra maps
Lotka-Volterra map Modifications	Lotka-Volterra map Modifications
Modification (i)	Modification (i)
 If 0 ≤ a < ⁴√1 + 4/√17 = 1.1847437 then P = [4 sin² π/17, 0] is a saddle fixed point of G⁴. If ⁴√1 + 4/√17 ≤ a ≤ 2 then P = [4 sin² π/17, 0] is not a saddle fixed point of G⁴. 	 If 0 ≤ a < ⁴√1 + 4/√17 = 1.1847437 then P = [4 sin² π/17, 0] is a saddle fixed point of G⁴. If ⁴√1 + 4/√17 ≤ a ≤ 2 then P = [4 sin² π/17, 0] is not a saddle fixed point of G⁴.

$$\label{eq:second} \begin{tabular}{|c|c|c|} \hline \begin{tabular}{|c|c|c|} \hline \begin{tabular}{|c|c|c|} \hline \begin{tabular}{|c|c|c|} \hline \begin{tabular}{|c|c|c|} \hline \begin{tabular}{|c|c|c|} \hline \begin{tabular}{|c|c|} \hline \begin{tabular$$

Lotka-Volterra map Modifications Lotka-Volterra map Modifications Modifications (ii) and (iii) Modifications (ii) and (iii) Let Let $G[x, y] = [x(4 - x - \varphi_x(y)), x\varphi_x(y)]$ $G[x, y] = [x(4 - x - \varphi_x(y)), x\varphi_x(y)]$ where where $\varphi_{x}(y) = \frac{\sqrt{2y(4-x) + x^{2}(4-x)^{2}} - \sqrt{y(4-x) + x^{2}(4-x)^{2}}}{\sqrt{2+x^{2}} - \sqrt{1+x^{2}}}$ $\varphi_{x}(y) = \frac{\sqrt{2y(4-x) + x^{2}(4-x)^{2}} - \sqrt{y(4-x) + x^{2}(4-x)^{2}}}{\sqrt{2+x^{2}} - \sqrt{1+x^{2}}}$ or or $\varphi_{x}(y) = \sqrt{(4-x)y} \; .$ $\varphi_{x}(y) = \sqrt{(4-x)y} \; .$ Then all lower fixed points different from [0,0] of the map G^n are Then all lower fixed points different from [0,0] of the map G^n are repulsive. In the case (ii) repulsive. In the case (ii) $\widetilde{\lambda}_2 > \left(rac{\sqrt{2}+1}{2} ight)'' > 1$. $\widetilde{\lambda}_2 > \left(rac{\sqrt{2}+1}{2} ight)'' > 1$. In the case (iii) the map G is not differentiable on the lower side. In the case (iii) the map G is not differentiable on the lower side. Peter Maličký Modified Lotka-Volterra maps Peter Maličký Modified Lotka-Volterra maps Lotka-Volterra map Lotka-Volterra map Modifications Modifications Modification (iv) Modification (iv) Let Let $G[x,y] = \begin{cases} [0,0] & \text{if } x = 0, \\ [x(4-x-\varphi_x(y)), x\varphi_x(y)] & \text{otherwise}. \end{cases}$ $G[x,y] = \begin{cases} [0,0] & \text{if } x = 0, \\ [x(4-x-\varphi_x(y)), x\varphi_x(y)] & \text{otherwise}. \end{cases}$ where where $\varphi_{x}(y) = (4-x)\left(\frac{y}{4-x}\right)^{2}.$ $\varphi_{X}(y) = (4-x) \left(\frac{y}{4-x}\right)^{\star} .$ Then any lower fixed point of the map G^n which is strictly Then any lower fixed point of the map G^n which is strictly repulsive for F^n is a strict saddle fixed point for G^n repulsive for F^n is a strict saddle fixed point for G^n and there exists an interior fixed point of G^n with the same period and itinerary.

Lotka-Volterra map Modifications Modifications

Modification (iv)

Let

$$G[x,y] = \begin{cases} [0,0] & \text{if } x = 0, \\ [x(4-x-\varphi_x(y)), x\varphi_x(y)] & \text{otherwise} \end{cases}$$

where

$$\varphi_x(y) = (4-x)\left(\frac{y}{4-x}\right)^x$$
.

Then any lower fixed point of the map G^n which is strictly repulsive for F^n is a strict saddle fixed point for G^n and there exists an interior fixed point of G^n with the same period and itinerary. Any lower fixed point of the map G^n which is a strict saddle fixed point for F^n is strictly repulsive for G^n .

> Peter Maličký Modified Lotka-Volterra maps

$G[x, y] = \begin{cases} [0, 0] & \text{if } x = 0, \\ [x(4 - x - \varphi_x(y)), x\varphi_x(y)] & \text{otherwise}, \end{cases}$ $\varphi_{\mathrm{x}}(\mathrm{y}) = (4-\mathrm{x})\left(\frac{\mathrm{y}}{4-\mathrm{x}}\right)^{\mathrm{x}}$. Then any lower fixed point of the map G^n which is strictly Peter Maličký Modified Lotka-Volterra maps Lotka-Volterra map Modifications F. Balibrea, J. L. G. Guirao, M. Lampart and J. Llibre. Dynamics of a Lotka-Volterra map. Fund. Math. 191 (2006), 265-279. J. L. G. Guirao and M. Lampart. Transitivity of a Lotka-Volterra map. Discrete Contin. Dyn. Syst. Ser. B **9** (2008), no. 1, 75-82 (electronic). P. Maličký. Interior periodic points of a LotkaVolterra map. J. Difference Equ. Appl. 18 (2012), no. 4, 553-567. A. N. Sharkovskii. Low dimensional dynamics. *Tagunsbericht* 20/1993, Proceedings of Mathematisches Forschunginstitut

G. Swirszcz. On a certain map of the triangle. *Fund. Math.* 155 (1998), 45-57.

Oberwolfach 1993. 17.

Modified Lotka-Volterra maps Peter Maličký

Modification (iv)

Let

where

repulsive for F^n is a strict saddle fixed point for G^n and there exists an interior fixed point of G^n with the same period and itinerary. Any lower fixed point of the map G^n which is a strict saddle fixed point for F^n is strictly repulsive for G^n .