Discrete Dynamics and Spectral Theory

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When dealing with nonautonomous difference equations, it is well-known that eigenvalues yield no information on the stability or hyperbolicity of linear systems. We therefore review several more appropriate spectral notions first.

Our particular focus is on the *dichotomy spectrum* (also called *dynamical* or *Sacker-Sell spectrum*). It is a crucial notion in the theory of dynamical systems, since it contains information on stability, as well as appropriate robustness properties. However, recent applications in nonautonomous bifurcation theory showed that a detailed insight into the fine structure of this spectral notion is necessary. On this basis, we explore a helpful connection between the dichotomy spectrum and operator theory. It relates the asymptotic behavior of linear nonautonomous difference equations to the point, surjectivity and Fredholm spectra of weighted shifts. This link yields several dynamically meaningful subsets of the dichotomy spectrum, which not only allows to classify and detect bifurcations, but also simplifies proofs for results on the long term behavior of difference equations with explicitly time-dependent right-hand side.

References

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