Homoclinic trajectories of non-autonomous maps

THORSTEN HÜLS

Department of Mathematics, Bielefeld University, POB 100131, D-33501 Bielefeld, Germany

E-mail address: huels@math.uni-bielefeld.de
URL: http://www.math.uni-bielefeld.de/~huels

For time-dependent dynamical systems of the form

$$x_{n+1} = f_n(x_n), \quad n \in \mathbb{Z}, \tag{1}$$

homoclinic trajectories are the non-autonomous analog of homoclinic orbits from the autonomous world, cf. [1]. More precisely, two trajectories $(x_n)_{n\in\mathbb{Z}}$, $(y_n)_{n\in\mathbb{Z}}$ of (1) are called homoclinic to each other, if

$$\lim_{n\to\pm\infty} \|x_n - y_n\| = 0.$$

Two boundary value problems are introduced, the solution of which yield finite approximations of these trajectories. Under certain hyperbolicity assumptions, we prove existence, uniqueness and error estimates.

Extending these ideas, we also propose adequate notions for heteroclinic orbits in non-autonomous systems, see [2].

The resulting algorithms and error estimates are illustrated by an example.

References

[1] Thorsten Hüls; *Homoclinic trajectories of non-autonomous maps*, J. Difference Equ. Appl., 17(1):9–31, 2011.

[2] Thorsten Hüls and Yongkui Zou; *On computing heteroclinic trajectories of non-autonomous maps*, Discrete Contin. Dyn. Syst. Ser. B 17(1):79–99, 2012.