Mandelpinski Necklaces for Singularly Perturbed Rational Maps

ROBERT L. DEVANEY

Department of Mathematics,
Boston University,
Boston, MA
USA
E-mail address: `bob@bu.edu`
URL: `http://math.bu.edu/people/bob`

In this lecture we consider rational maps of the form $z^n + C/z^n$ where $n > 2$. When C is small, the Julia sets for these maps are Cantor sets of circles and the corresponding region in the C-plane (the parameter plane) is the McMullen domain. We shall show that the McMullen domain is surrounded by infinitely many simple closed curves called Mandelpinski necklaces. The kth necklace contains exactly $(n-2)n^k + 1$ parameters that are the centers of baby Mandelbrot sets and the same number of parameters that are centers of Sierpinski holes, i.e., disks in the parameter plane where the corresponding Julia sets are Sierpinski curves (sets that are homeomorphic to the Sierpinski carpet fractal). We shall also briefly describe other interesting structures in the parameter plane.