Oscillation theory of discrete symplectic systems with nonlinear dependence on the spectral parameter

ROMAN ŠIMON HILSCHER

Department of Matematics and Statistics, Faculty of Science, Masaryk University
Kotlářská 2, CZ-61137 Brno, Czech Republic

E-mail address: hilscher@math.muni.cz
URL: http://www.math.muni.cz/~hilscher

Symplectic systems represent a discrete time analogue of the linear Hamiltonian systems. They contain as special cases many important difference equations and systems, namely the Sturm–Liouville difference equations, symmetric three-term recurrence equations, Jacobi difference equations, and linear Hamiltonian difference systems. Following our recent work in [3] and [2], we introduce a new theory of discrete symplectic systems, in which the dependence on the spectral parameter is nonlinear. This requires to develop new definitions of (finite) eigenvalues and (finite) eigenfunctions and their multiplicities for such systems. Our main results include the corresponding oscillation theorems, which relate the number of (finite) eigenvalues with the number of focal points of the principal solution in the given discrete interval. The present theory generalizes several known results for discrete symplectic systems which depend linearly on the spectral parameter, such as in [1]. We also show that our results are new even for the above mentioned special discrete symplectic systems.

References

[1] O. Došlý, W. Kratz, *Oscillation theorems for symplectic difference systems*, J. Difference Equ. Appl. **13** (2007), no. 7, 585–605.

[2] W. Kratz, R. Šimon Hilscher, *A generalized index theorem for monotone matrix-valued functions with applications to discrete oscillation theory*, in preparation (2012).

[3] R. Šimon Hilscher, *Oscillation theorems for discrete symplectic systems with nonlinear dependence in spectral parameter*, submitted (2012).