Existence, uniqueness and attractivity of prime period two solution for a difference equation of exponential form

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In [1] the authors studied the existence of the equilibrium and the boundedness of solutions of the difference equation

$$x_{n+1} = a + bx_{n-1}e^{-x_n} \tag{1}$$

where a, b are positive constants and the initial values x_{-1}, x_0 are positive numbers. Moreover the authors gave a conjecture concerning the existence, the uniqueness and the attractivity of prime period two solution.

In this paper we give an answer concerning the existence and the uniqueness of a prime period two solution for the equation (1). Moreover we find solutions of (1) which converge to the unique periodic solution of period two.

Equation (1) may have applications in Biology if we consider a as the immigration rate and b as the population growth rate.

References

[1] E. El-Metwally, E.A. Grove, G. Ladas, R. Levins, M. Radin, On the difference equation, $x_{n+1} = \alpha + \beta x_{n-1}e^{-x_n}$, Nonlinear Analysis 47 (2001) 4623-4634.