On the existence of a weighted asymptotically constant solutions of Volterra difference equations of nonconvolution type

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We study a Volterra difference equation of the form $x(n+1) = a(n) + b(n)x(n) + c(n)x(n-1) + \sum_{i=0}^{n} K(n,i)x(i)$ where $n \in \mathbb{Z}$, $a, b, c, x\colon \mathbb{Z} \to \mathbb{R}$ and $K\colon \mathbb{Z} \times \mathbb{Z} \to \mathbb{R}$. For every admissible constant $c^* \in \mathbb{R}$, sufficient conditions for the existence of a solution $x = x(n)$ of the above equation such that $x(n) \sim c^* n\beta(n)$, where $\beta(n) = \frac{1}{2^n} \prod_{j=0}^{n-1} b(j)$ are presented. Next, sufficient conditions for the existence of an eventually positive, oscillatory, and quickly oscillatory solution of this equation are obtained, as a corollary of the main result. Finally, a conditions under which considered equation possesses an asymptotically periodic solution are given.

References

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