Admissibility of linear stochastic discrete Volterra operators applied to an affine stochastic convolution equation

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The long run behaviour of a finite dimensional finite delay summation equation with an additive Gaussian noise is analysed. This equation may be considered as a generalisation of an autoregressive process of arbitrary, but finite, order. It is known from existing theory that the asymptotic behaviour of the resolvent function of such an equation may be expressed in terms of the roots of its characteristic equation, c.f. e.g. [1]. It is shown that the solution of the stochastic equation is also reliant upon the leading order roots of the characteristic equation.

Admissibility theory of deterministic equations has been studied in connection with the asymptotic theory of such equations, in e.g. [2]. The authors develop a stochastic admissibility theory of linear Volterra operators to obtain their results. While the asymptotic results described in this presentation hold almost surely it is shown that this mode of convergence implies convergence in mean square.

In addition to the finite delay stochastic equation, a Volterra summation equation is also discussed. Some examples of the results are sketched which illustrate the differing types of long run behaviour which may occur depending upon the order of the leading roots and whether the leading roots are purely real or complex.

References

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