Studying the Influence of Higher Order Derivative Corrections in String Theory Problems by means of Computer Algebra

Valentina Kolybasova
Faculty of Physics, Department of Mathematics
Lomonosov Moscow State University
Moscow, Russia

Abstract

The low-energy limit of string gravity in the semi-classical approximation is investigated in collaboration with Dr. M. V. Pomazanov and Dr. S. O. Alexeyev. The method of singularly perturbed ordinary differential equations is used. The most simple spherically symmetric static four dimensional metric is studied:

$$ds^2 = -\Delta(r)dt^2 + \frac{\sigma^2(r)}{\Delta(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right).$$

In this particular case, the Lagrangian is made of power series of the string coupling constant, and has the form

$$\begin{aligned}\mathcal{L} &= L(r,\Delta,\sigma,\phi,\Delta',\phi') + \lambda L_1(r,\Delta,\sigma,\phi,\Delta',\phi') + \\ &\quad + \lambda^n l(r,\Delta,\sigma,\phi,\Delta',\sigma',\Delta'') + \text{higher order corrections},\end{aligned}$$

where Δ, σ, ϕ are functions of r, ϕ is a dilatonic field, λ is the string coupling constant, $n = 2$ or $n = 3$ depending on which variant of low-energy string theory is chosen: bosonic, heterotic or SUSY II. Exact form of L_1 and l also depends on this choice. The unperturbed Lagrangian L yields asymptotically flat spherically symmetric static black hole solutions (the Schwarzschild metric):

$$\Delta = \Delta^0(r) = 1 - \frac{2M}{r}, \qquad \sigma = \sigma^0(r) = 1, \quad \phi = \phi^0(r) = \phi_0 = \text{const},$$

where M is the mass of the black hole.

The problem of proximity between the solution to the Lagrangian equations and the corresponding solution in classical gravity is extremely important. Since these equations are singularly perturbed, with higher derivatives of metric in the Lagrangian, their solutions might be fundamentally different from unperturbed ones. The low-energy limit must be considered carefully to avoid any internal contradiction in the theory.

A great amount of calculations is carried out, due to the complexity and unhandiness of the corresponding corrections. So, analytical and numerical calculations on a computer are performed. Software packages MAPLE and REDUCE are used to derive

calculations are performed. Then, proximity of the perturbed solution to the unperturbed one with the same initial conditions for three different forms of the Lagrangian is studied using Tikhonov's theorem. The necessary condition of the pointwise proximity is $u(r) \le 0$ if $r \ge 0$ for all solutions of the equation

$$|uP - Q|_{\Delta=\Delta^0(r),\sigma=\sigma^0(r),\phi=\phi^0(r)} = 0,$$

where

$$Q = \begin{pmatrix} L_{\Delta'\Delta'} - L^2_{\Delta'\phi'}/L_{\phi'\phi'} & L_{\sigma\Delta'} - L_{\sigma\phi'}L_{\phi'\Delta'}/L_{\phi'\phi'} \\ L_{\Delta'\sigma} - L_{\Delta'\phi'}L_{\phi'\sigma}/L_{\phi'\phi'} & L_{\sigma\sigma} - L^2_{\sigma\phi'}/L_{\phi'\phi'} \end{pmatrix},$$

$$P = \begin{pmatrix} l_{\Delta''\Delta''} & l_{\Delta''\sigma'} \\ l_{\Delta''\sigma'} & l_{\sigma'\sigma'} \end{pmatrix}.$$

For the sake of errors elimination, two different codes are written, one for MAPLE and the other for REDUCE, and all the results agreed.

This approach led to serious and unexpected conclusions. It is demonstrated that the correction to the Lagrangian, in two different variants of low-energy string gravity (bosonic and heterotic), makes a singular contribution to the equations, because the condition $u \le 0$ is not fulfilled. The proximity of this solution to the classical one is possible only with a particular choice of additional initial conditions, which could hardly be based on the physics.

In SUSY II case, the necessary proximity condition holds. However, a sufficient condition is not obtained analytically, because it demands taking into account the next correction to the Lagrangian. The seventh order Runge-Kutta code is used for the numerical integration of Lagrangian equations in SUSY II case. The initial conditions at $r = 2.5M$ are derived from the unperturbed solution $\Delta^0(r)$, $\sigma^0(r)$, $\phi^0(r)$. Numerical analysis revealed that the correct behavior of the solution is observed without any fine-tuning of initial conditions. So, from this point of view, SUSY II seems to be the most promising variant of the theory. This fact is extremely important for the specialists in this field as the experimental testing of the string gravity consequences should be based on an internally consistent model.