The period of the limit cycle bifurcating from a persistent polycycle

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June 21, 2023

Abstract. We consider smooth families of planar polynomial vector fields $\{X_\mu\}_{\mu\in\Lambda}$, where Λ is an open subset of $\mathbb{R}^N$, for which there is a hyperbolic polycycle Γ that is persistent (i.e., such that none of the separatrix connections is broken along the family). It is well known that in this case the cyclicity of Γ at μ_0 is zero unless its graphic number $r(\mu_0)$ is equal to one. It is also well known that if $r(\mu_0)=1$ (and some generic conditions on the return map are verified) then the cyclicity of Γ at μ_0 is one, i.e., exactly one limit cycle bifurcates from Γ. In this paper we prove that this limit cycle approaches Γ exponentially fast and that its period goes to infinity as $1/|r(\mu)-1|$ when $\mu\to\mu_0$. Moreover, we prove that if those generic conditions are not satisfied, although the cyclicity may be exactly 1, the behavior of the period of the limit cycle is not determined.

1 Introduction and main results

This work deals with the study of the period of limit cycles arising in bifurcations for families of smooth planar vector fields $\{X_\mu\}_{\mu\in\Lambda}$, where $\Lambda\subset\mathbb{R}^N$. From the classification of first-order structurally unstable vector fields (see, for instance [1, 6, 11]), the generic compact isolated bifurcations for one-parameter (i.e., $N=1$) families of smooth planar vector fields which give rise to periodic orbits for $\mu\to\mu_0$ are: the Andronov-Hopf bifurcation, the bifurcation of a semi-stable periodic orbit, the saddle-node loop and the saddle loop bifurcations. These are referred to as the *elementary bifurcations*. In [4] the authors determined the behavior of the period $\mathscr{T}(\mu)$ of the limit cycle of X_μ arising from a elementary bifurcation as $\mu\to\mu_0$. More precisely, they obtained the principal term of the expression of $\mathscr{T}(\mu)$ which is comprised in the following list:

(*i*) $\mathscr{T}(\mu)\sim T_0+T_1(\mu-\mu_0)$ for the Andronov-Hopf bifurcation;

(*ii*) $\mathscr{T}(\mu)\sim T_0+T_1\sqrt{|\mu-\mu_0|}$ for the bifurcation from a semi-stable periodic orbit;

(*iii*) $\mathscr{T}(\mu)\sim T_0/\sqrt{|\mu-\mu_0|}$ for the saddle-node loop bifurcation;

2010 *AMS Subject Classification*: 34C07; 34C20; 34C23.
Key words and phrases: limit cycle, polycycle, cyclicity, period, asymptotic expansion, Dulac map.