PHASE PORTRAITS OF THE COMPLEX RICCATI EQUATION WITH CONSTANT COEFFICIENTS

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ABSTRACT. In this paper we characterize the phase portrait of the complex Riccati quadratic polynomial differential systems

$$\frac{dz}{dt} = \dot{z} = a(z - b)(z - c),$$

with $z \in \mathbb{C}$, $a, b, c \in \mathbb{C}$ with $a \neq 0$ and $t \in \mathbb{R}$. Taking $z = x + iy$, and writing the Riccati equation as the differential system $(\dot{x}, \dot{y})$ in the plane, we give the complete description of their phase portraits in the Poincaré disk (i.e. in the compactification of $\mathbb{R}^2$ adding the circle $\mathbb{S}^1$ of the infinity) modulo topological equivalence.

1. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

Numerous problems of applied mathematics are modeled by quadratic polynomial differential systems. Excluding linear systems, such systems are the ones with the lowest degree of complexity, and the large bibliography on the subject proves its relevance. We refer for example to the books of Ye Yanqian et al. [15], Reyn [13], and Artes, Llibre, Schlomiuk, Vulpe [1], and the surveys of Coppel [5], and Chicone and Jinghuang [4] are excellent introductory readings to the quadratic polynomial differential systems.

Among such quadratic polynomial differential systems we emphasize the Riccati polynomial systems. Since their introduction at the end of the seventeenth century with the crucial work of Jacobo F. Riccati based on the variable separable technique, they have been studied intensively by renown mathematicians with numerous approaches underlying all the facets of their richness. Their field of applications is widespread ranging from continued fractions to some useful applications in control system theory, see for instance [8, 11, 12, 14].

In this paper we characterize the phase portraits of the complex Riccati differential equation

$$(1) \qquad \dot{z} = a(z - b)(z - c)$$

with $z \in \mathbb{C}$, $a, b, c \in \mathbb{C}$ with $a \neq 0$ and the dot means derivative with respect to $t \in \mathbb{R}$. We write

$$z = x + iy, \quad a = a_1 + ia_2, \quad b = b_1 + ib_2, \quad c = c_1 + ic_2$$