4-DIMENSIONAL ZERO HOPF BIFURCATION OF QUADRATIC POLYNOMIAL DIFFERENTIAL SYSTEMS, VIA AVERAGING THEORY

DJAMILA DJEDID[1], JAUME LLIBRE [2] AND AMAR MAKHLOUF[1]

Abstract. It is known that the maximum number of limit cycles that can bifurcate from a zero-Hopf equilibrium point of a quadratic polynomial differential system in dimension two is 3, and that in dimension three is at least 3. Here we prove that in dimension 4 at least 9 limit cycles can bifurcate in a zero-Hopf bifurcation of a quadratic polynomial differential system.

1. Introduction and statement of the main result

A Hopf bifurcation takes place at a singular point of a differential system when this changes its stability. More precisely, it is a local bifurcation which can appears when a singular point of a differential system having a pair of complex conjugate eigenvalues crosses the imaginary axis of the complex plane when we move the parameters of the differential system. At this crossing under convenient assumptions on the differential system, one or several small-amplitude limit cycles bifurcate from the singular point.

When the pair of complex eigenvalues are on the imaginary axis, i.e. they are of the form $\pm bi$, if the other eigenvalues are non-zero, we talk about a *Hopf bifurcation*, but if some of the other eigenvalues are zero, we say that we have a *zero-Hopf bifurcation*. Here we are interested in the study of the zero-Hopf bifurcations when all the eigenvalues different from the $\pm bi$ are zero, we denote such kind of zero-Hopf bifurcation a *complete zero-Hopf bifurcation*. While there is a well developed theory for studying the Hopf bifurcations (see for instance [4, 10]), such theory does not exist for the zero-Hopf bifurcations. For the zero-Hopf bifurcations there are only partial results.

The goal of this paper is to study how many small-amplitude limit cycles can bifurcate in a complete zero-Hopf bifurcation at a singular point of a quadratic polynomial differential system in function of the dimension of the system.

Bautin [1] in 1954 proved that at most 3 small-amplitude limit cycles can bifurcate in a Hopf bifurcation at a singular point of a quadratic polynomial differential system in $\mathbb{R}^2$. Note that in $\mathbb{R}^2$ the notions of Hopf bifurction, zero-Hopf bifurcation and complete zero-Hopf bifurcation coincide.

2010 *Mathematics Subject Classification.* Primary: 34C07, 34C05, 34C40.

Key words and phrases. zero-Hopf bifurcation, averaging theory, quadratic polynomial differential systems.