FLEXIBILITY OF ENTROPIES FOR PIECEWISE EXPANDING UNIMODAL MAPS

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ABSTRACT. We investigate the *flexibility* of the entropy (topological and metric) for the class of piecewise expanding unimodal maps. We show that the only restrictions for the values of the topological and metric entropies in this class are that both are positive, the topological entropy is at most $\log 2$, and the metric entropy is not larger than the topological entropy.

In order to have a better control on the metric entropy, we work mainly with topologically mixing piecewise expanding skew tent maps, for which there are only 2 different slopes. For those maps, there is an additional restriction that the topological entropy is larger than $\frac{1}{2}\log 2$.

Moreover, we generalize and give a different interpretation of the Milnor-Thurston formula connecting the topological entropy and the kneading determinant for unimodal maps.

1. INTRODUCTION

Recently an important program in Dynamical Systems was initiated by Anatole Katok. It concerns *flexibility*, that is, the idea that for a given class of dynamical systems, dynamical invariants (for instance entropies) can take arbitrary values, subject only to natural restrictions. Various results in this direction were obtained for instance in papers [E, EK, BKRH].

Here we investigate the family of piecewise expanding unimodal maps. While they are not smooth, they are piecewise smooth (in fact, the maps that we consider are piecewise linear). For those maps, by [LaY], there exists an absolutely continuous invariant probability measure. By [LiY], this measure is unique. Therefore we can consider its metric entropy (which is also equal to its Lyapunov exponent), as well as the topological entropy of the map. Both entropies are positive, topological entropy is at most $\log 2$, and by the Variational Principle, the metric entropy is not larger than the topological entropy. We will show (Theorem B) that those are the only restrictions for the values of those entropies.

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