The Sierpinski Mandelbrot Spiral

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Boston University

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Outline

1. Introduction
2. $z^2 + \lambda/z^3$
3. $z^4 + \lambda/z^3$

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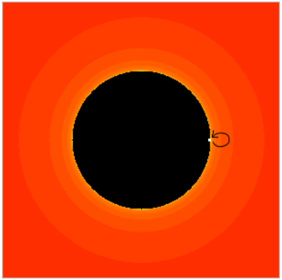

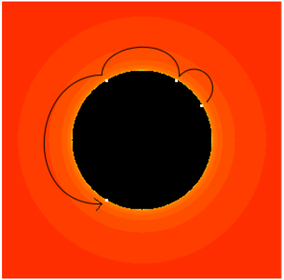

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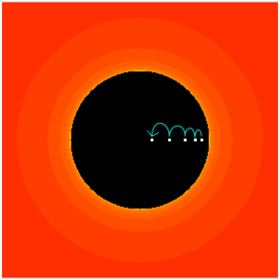

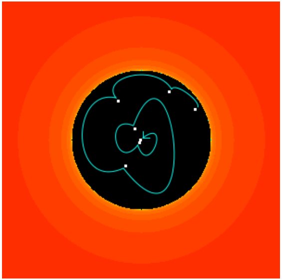

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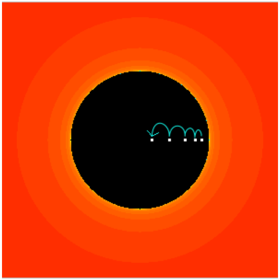
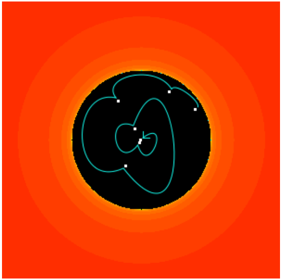

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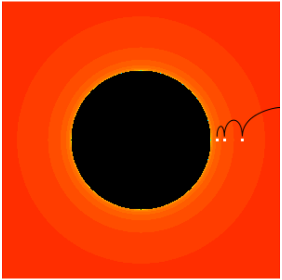
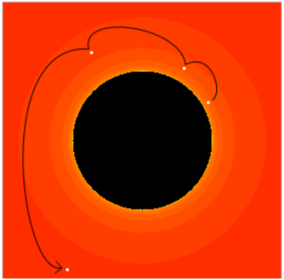

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- Points outside the circle stay outside the circle while approaching ∞, an attracting fixed point on the Riemann sphere.

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- the Fatou Set, or $\mathcal{F}(F)$. This is the complement of $\mathcal{J}(F)$ in the Riemann sphere.

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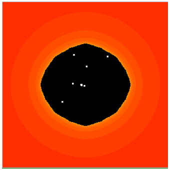

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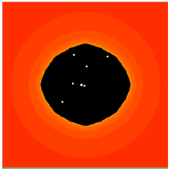

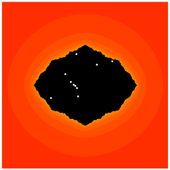

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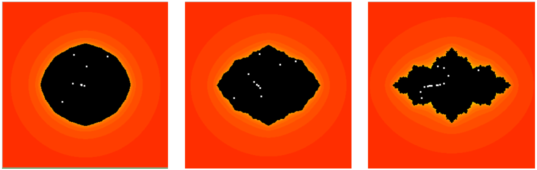

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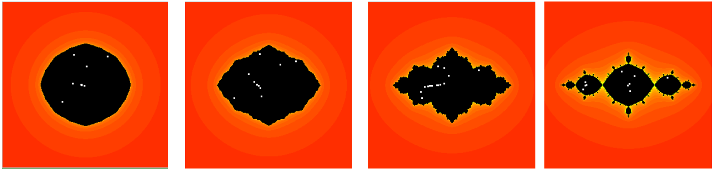

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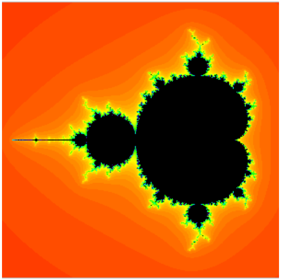

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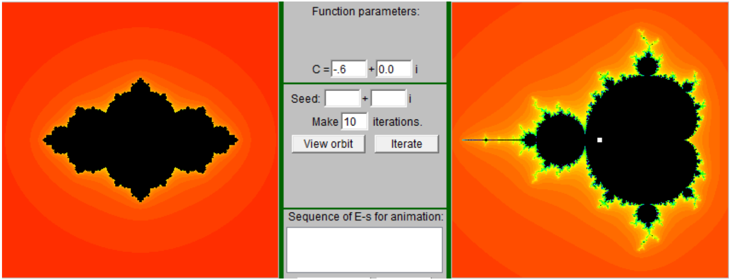
$\mathcal{J}(F_\lambda)$ depends on λ



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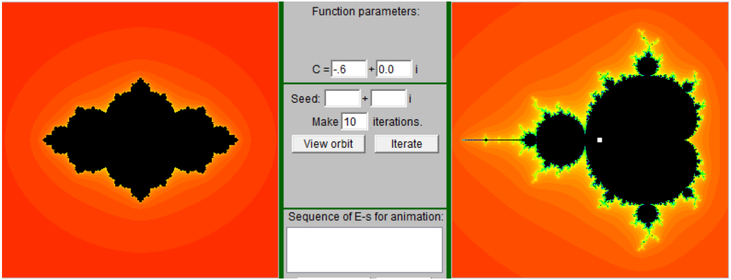

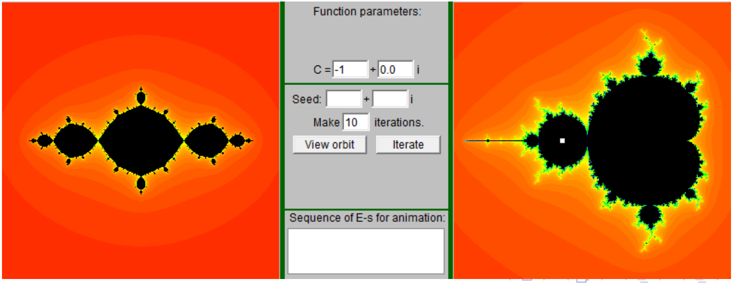

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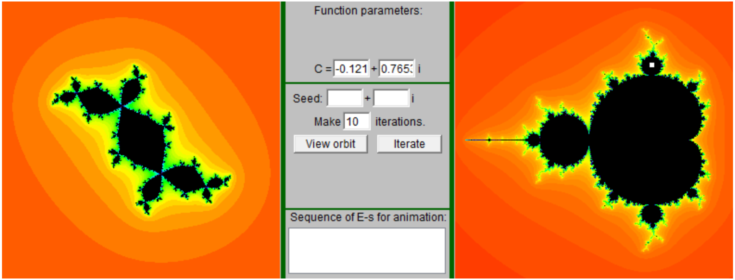

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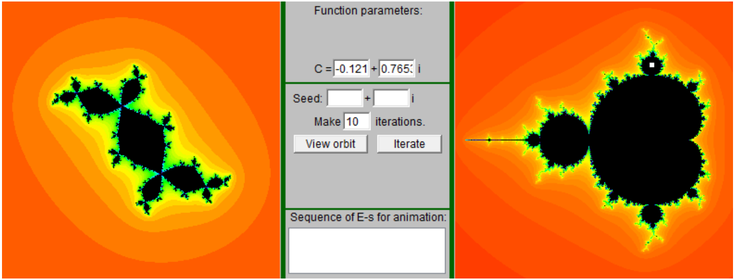

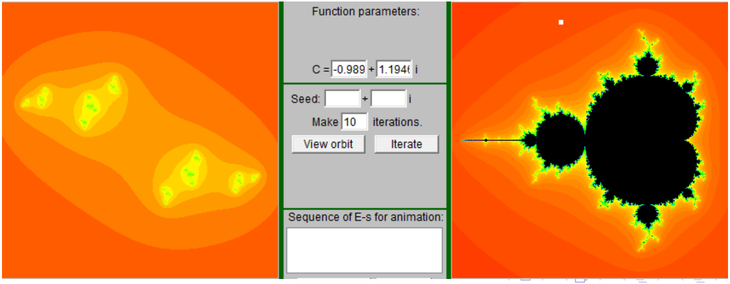

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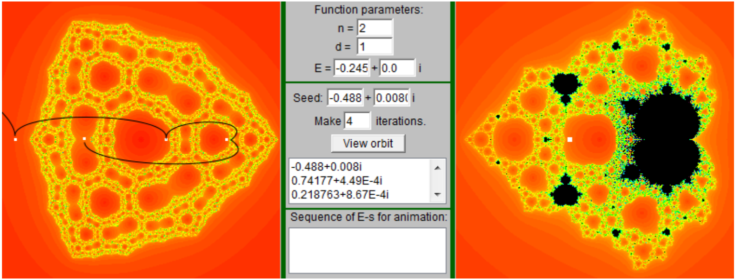

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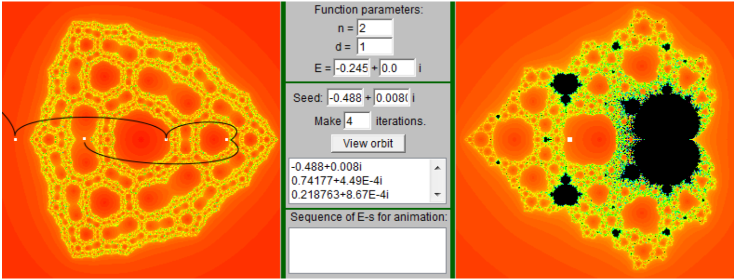
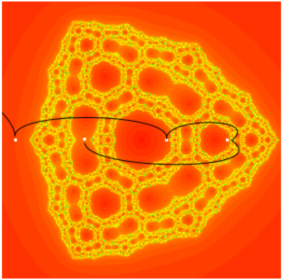


- This is the rational map with $n = 2, d = 1$.

The trap door and the basin



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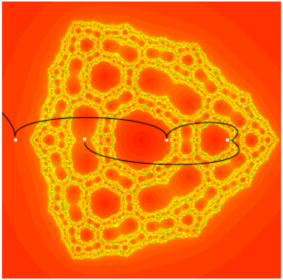

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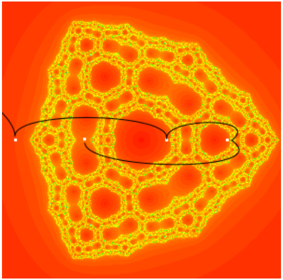

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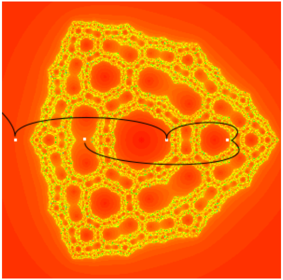

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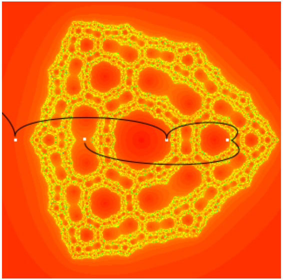

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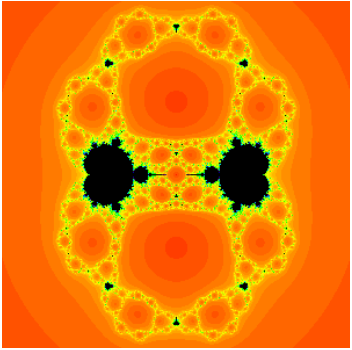

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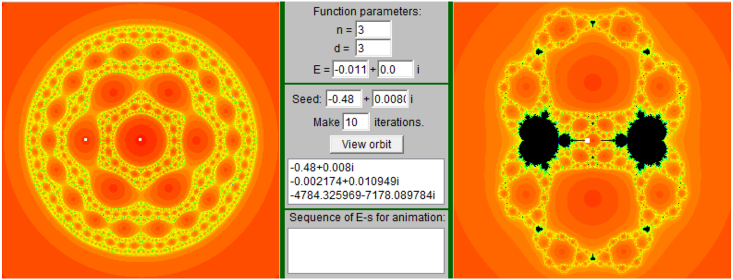

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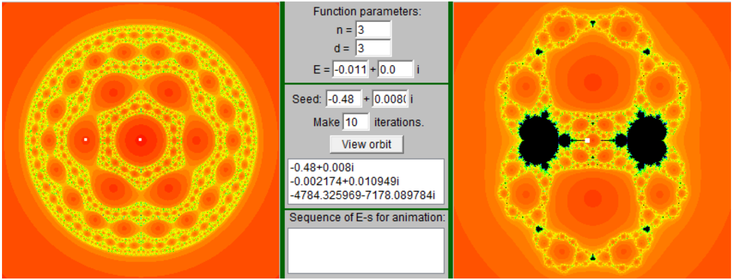
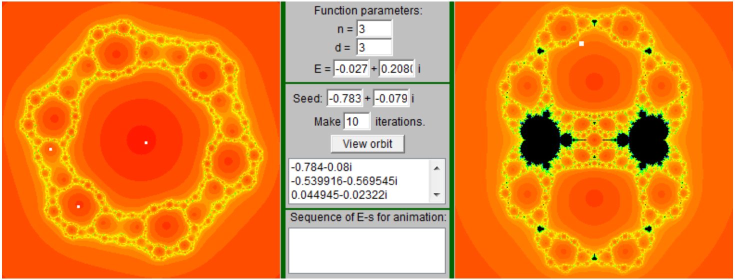


- The McMullen domain is the set of λ around the origin for which the critical point enters the trap door after 1 iteration.

Sierpinski holes



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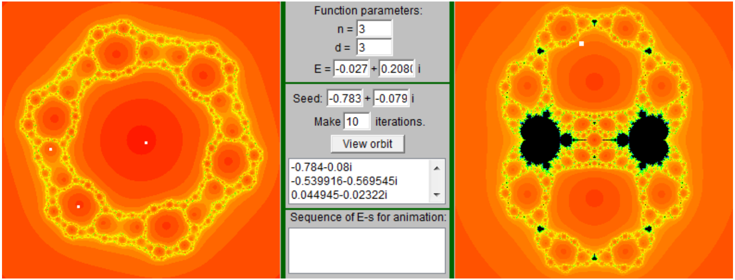

- A Sierpinski hole is a set of λ for which the critical point enters the trap door after 2 or more iterations.

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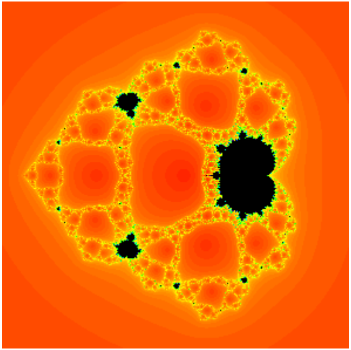

Critical points, critical values, and prepoles

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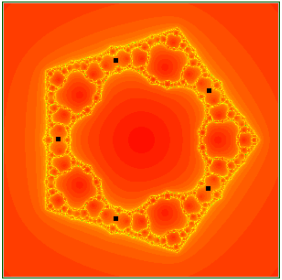

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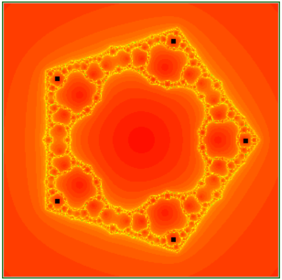

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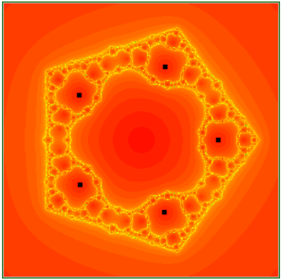
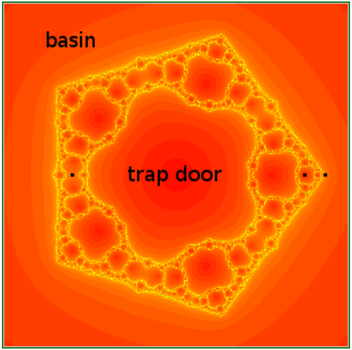


The Escape Trichotomy



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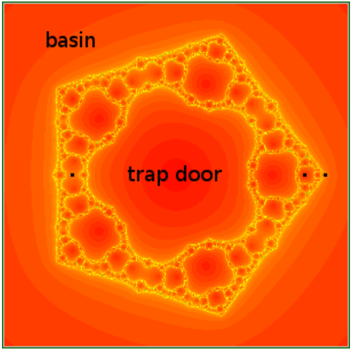

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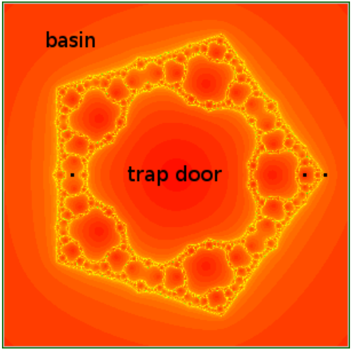

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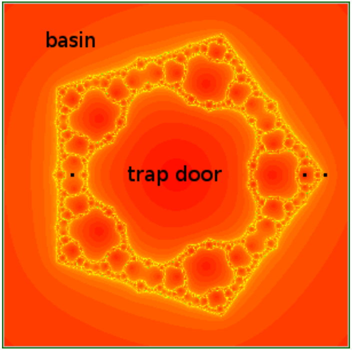

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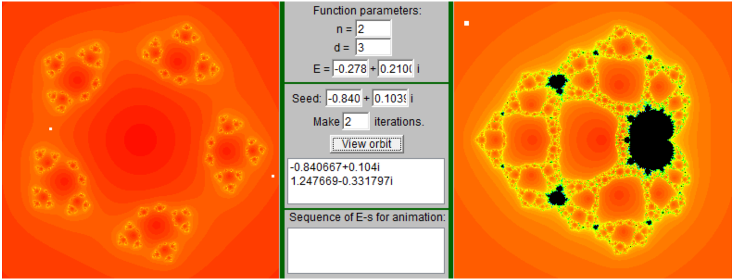
- We can classify the regions in the parameter plane by the orbit of that critical value.

Cantor set locus



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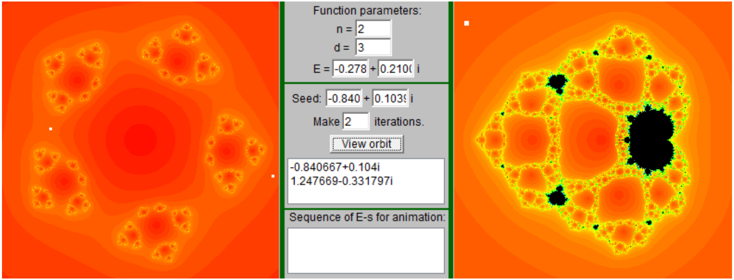
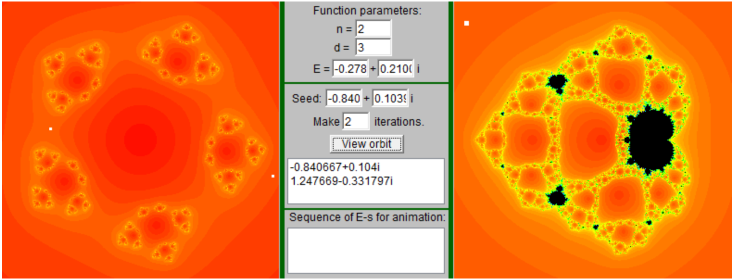


- v^{λ} lies in B_{λ}. In this case it is known that $\mathcal{J}(F_{\lambda})$ is a Cantor set.

Cantor set locus



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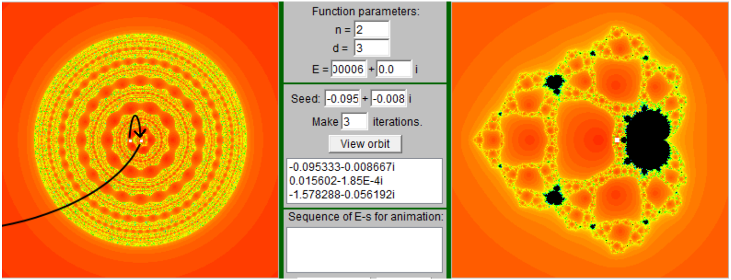
- The corresponding set of λ-values in the parameter plane is called the Cantor set locus.

McMullen domain



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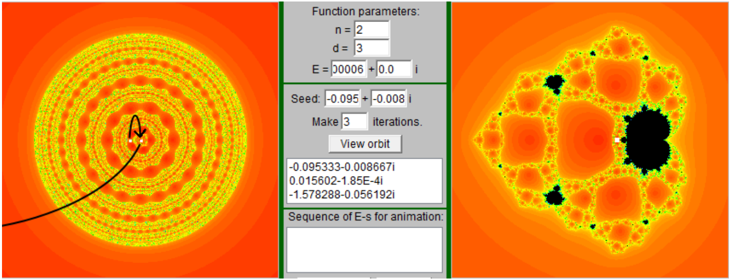

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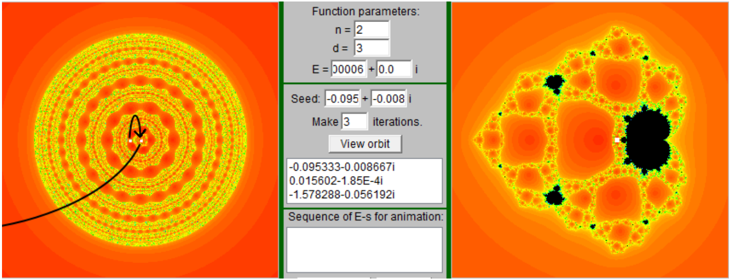

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- If you take a slice of the Julia set, you can kind of see the Cantor set in that interval.

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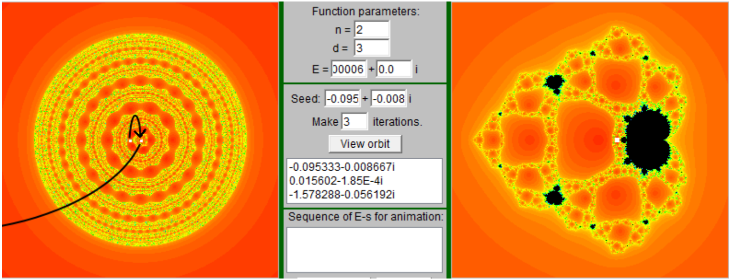

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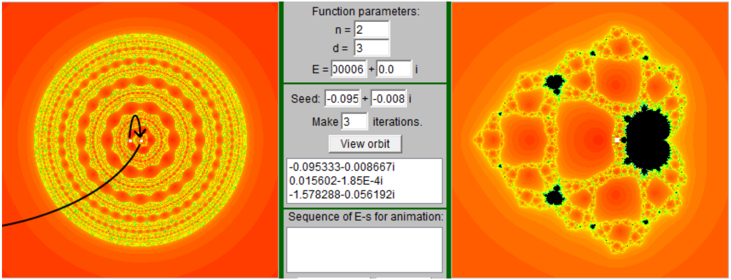

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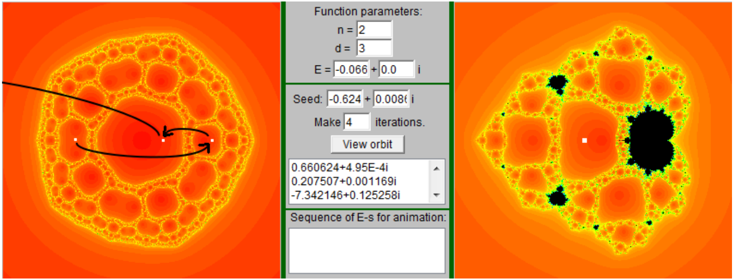
Sierpinski holes



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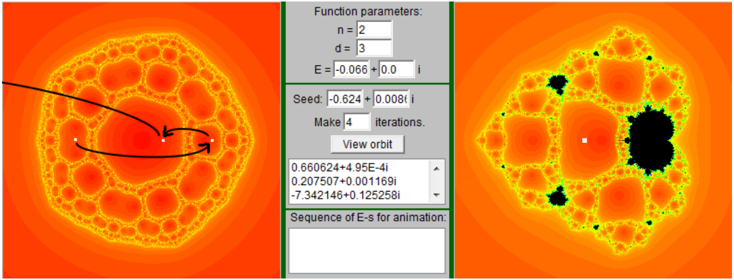
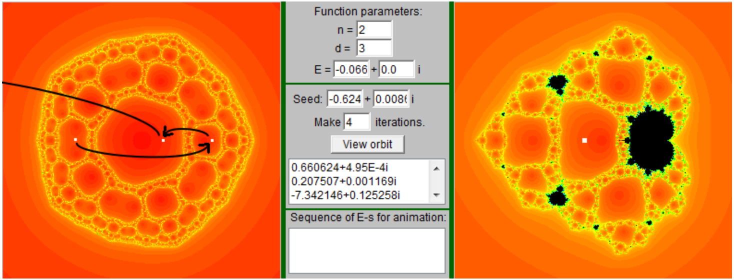


- c^λ enters T_λ at iteration 2.

Sierpinski holes



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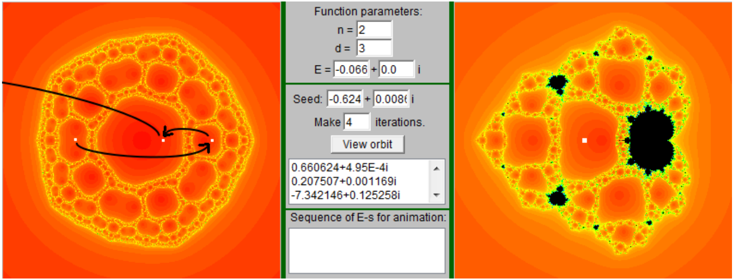
Sierpinski holes



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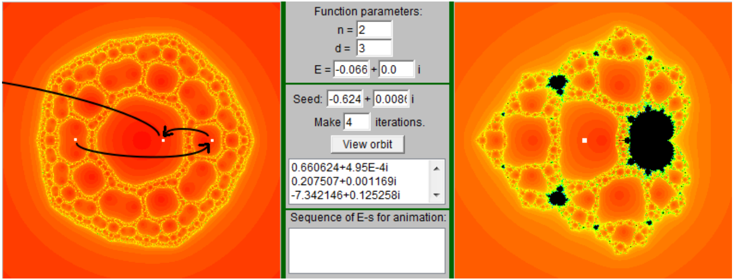

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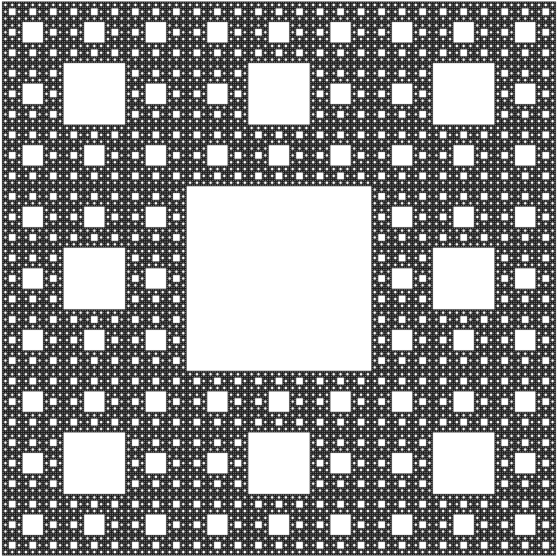
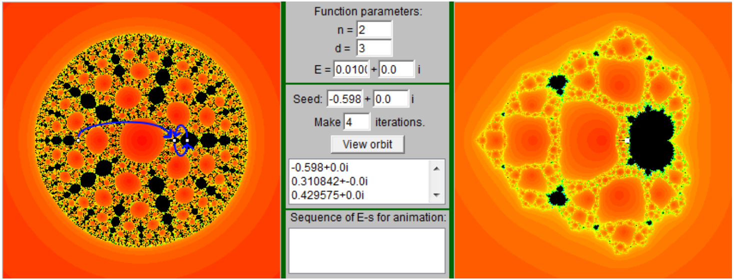
- The corresponding sets of λ are called Sierpinski holes.

Sierpinski carpet fractal



Mandelbrot sets and the connectedness locus



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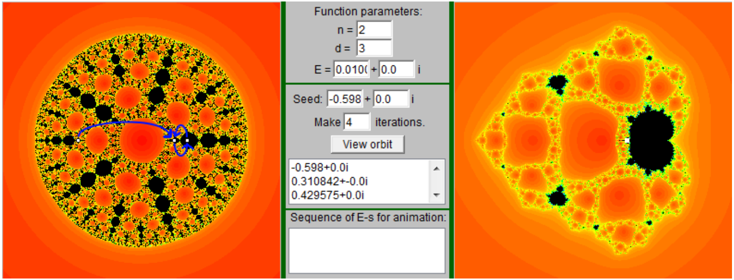

- v^λ does not escape to ∞.

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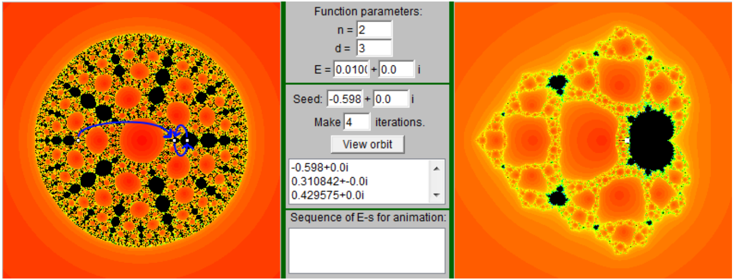

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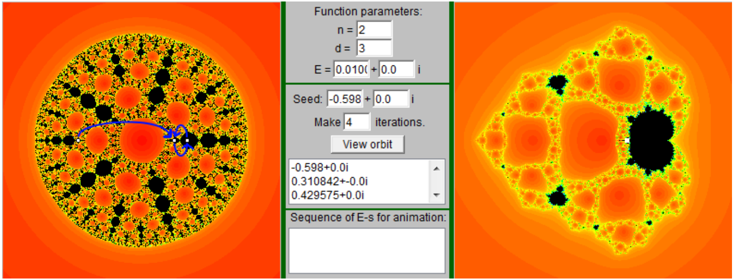

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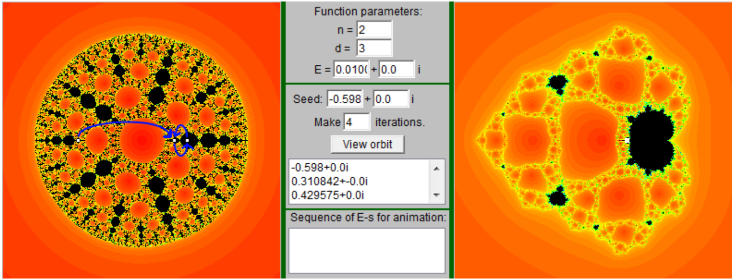

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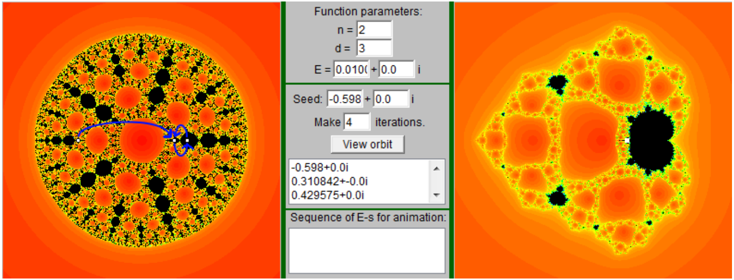

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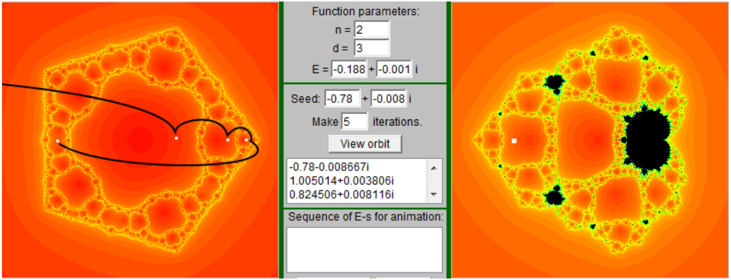
Sierpinski hole of higher escape time



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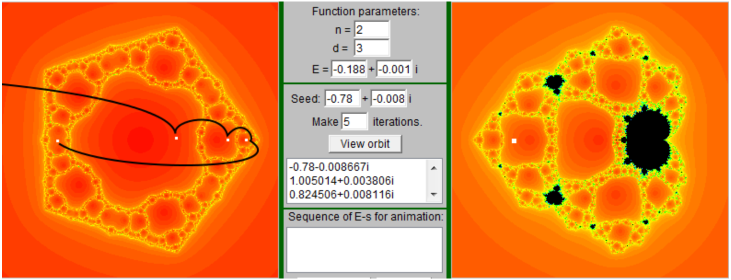
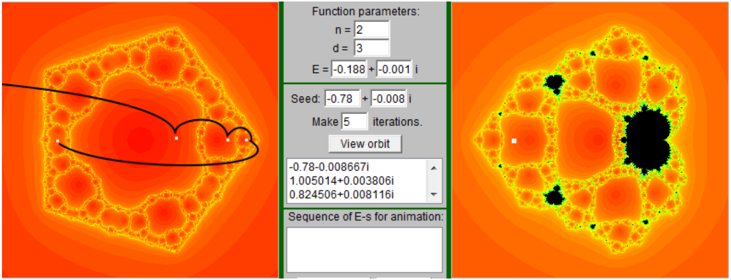


- For a λ in the next Sierpinski hole to the left:

Sierpinski hole of higher escape time



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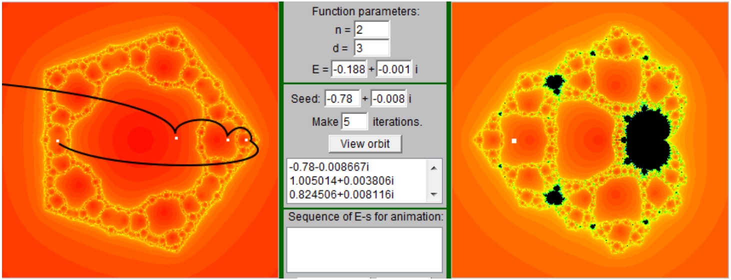
Sierpinski hole of higher escape time



- For a λ in the next Sierpinski hole to the left:
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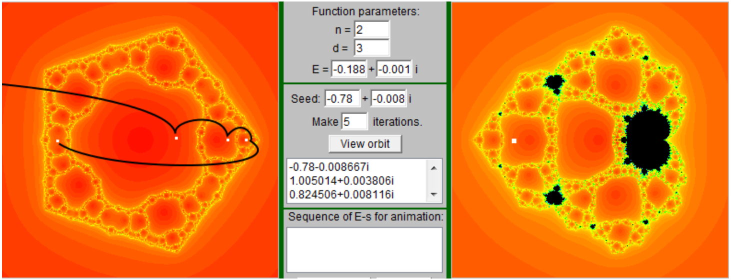
- The next Sierpinski hole along the negative real axis probably has escape time 4.

Sierpinski hole of higher escape time



- For a λ in the next Sierpinski hole to the left:
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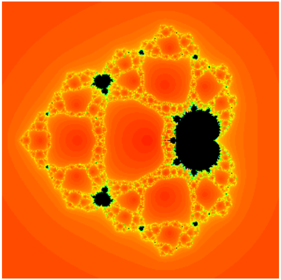
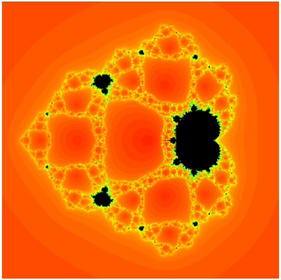
- This idea of increasingly higher escape time Sierpinski holes might be interesting...

More Mandelbrot sets

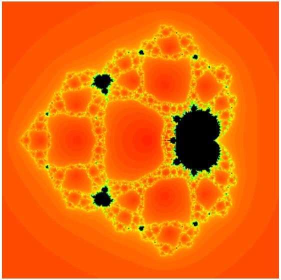


More Mandelbrot sets



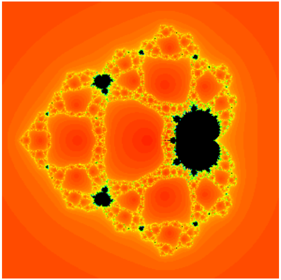
- There is the clearly visible principal Mandelbrot set.

More Mandelbrot sets



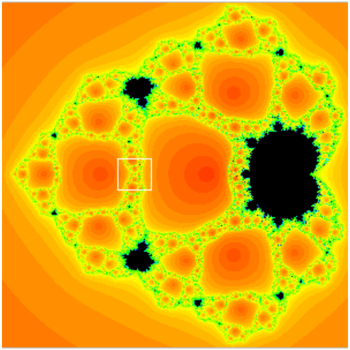
- There is the clearly visible principal Mandelbrot set.
- Also two baby Mandelbrot sets.

More Mandelbrot sets



- There is the clearly visible principal Mandelbrot set.
- Also two baby Mandelbrot sets.
- Six more baby Mandelbrot sets. Are there more?

Why yes there are



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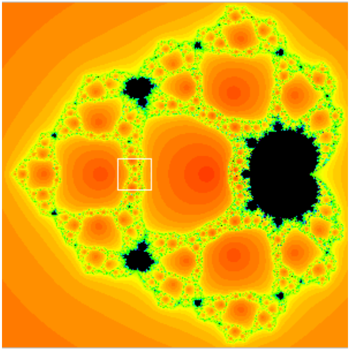

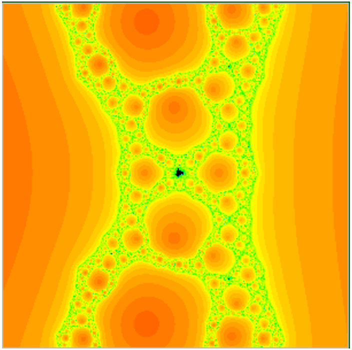

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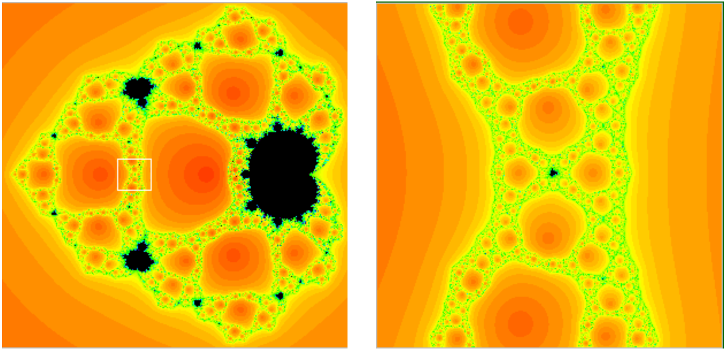
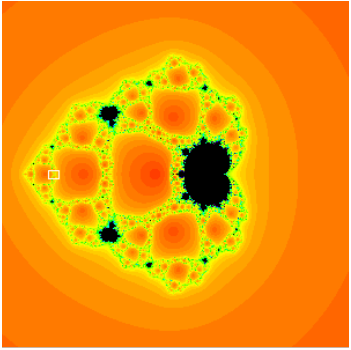


- There is a Mandelbrot between the Sierpinski holes of c^λ escape time 2 and 3.

Further along the negative real axis



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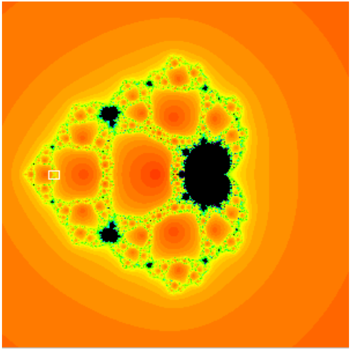

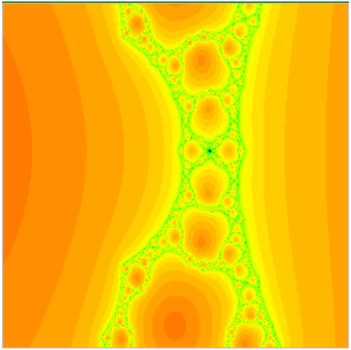

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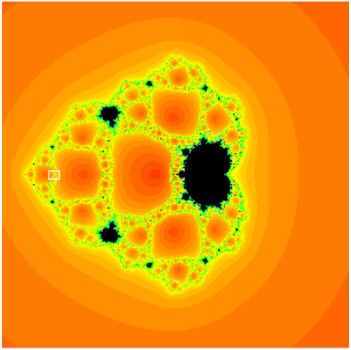

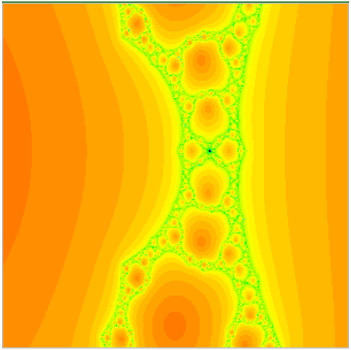

- Looks like another Mandelbrot set between the next pair of Sierpinski holes.

Claim

- There are infinitely many Sierpinski holes along the negative real axis of the parameter plane.

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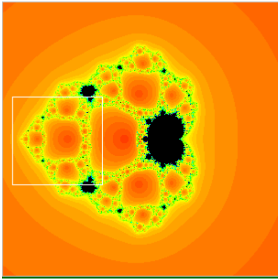

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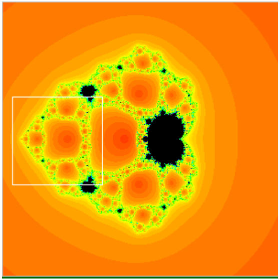

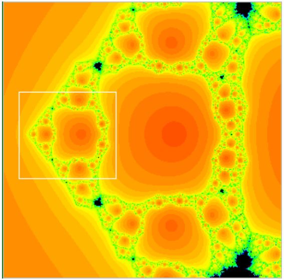

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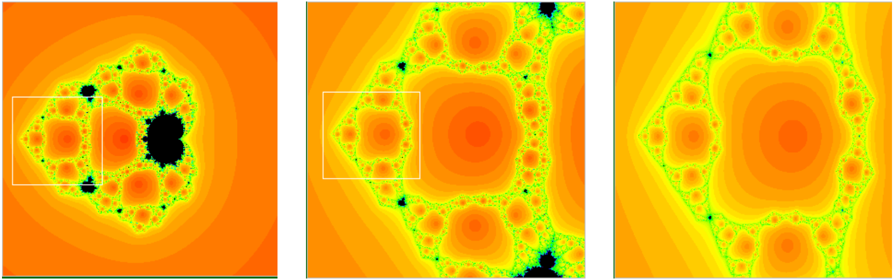

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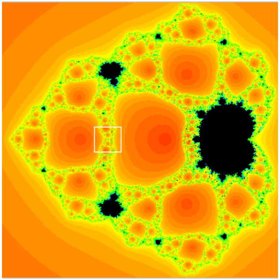

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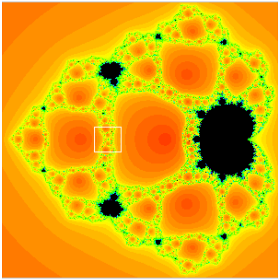

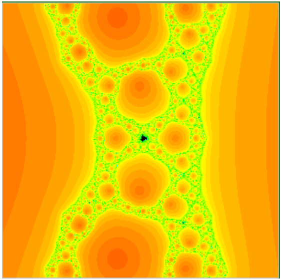

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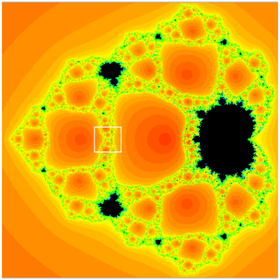
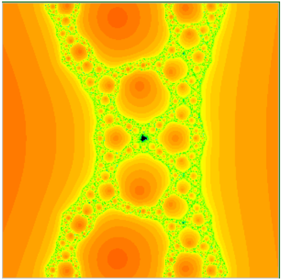
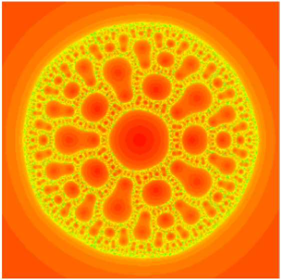




- This set of infinitely many alternating Sierpinski holes and Mandelbrot sets along the negative real axis in the parameter plane is the *Sierpinski Mandelbrot arc*.

The dynamical plane for $n = 2, d = 3$



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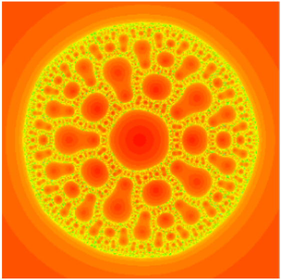

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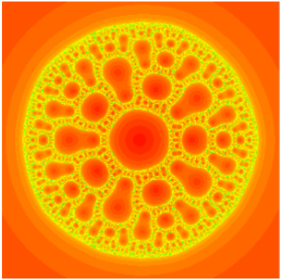

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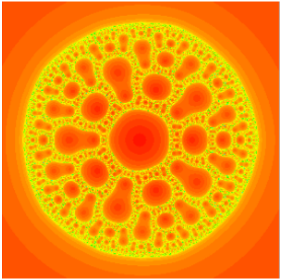

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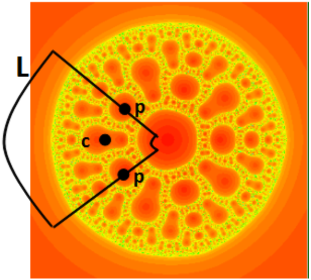
- To prove the existence of the Sierpinski Mandelbrot arc we will consider some closed sets in the dynamical plane.
- We will also restrict λ to an annular region in the parameter plane. The details are not that interesting. more

The left wedge L^λ



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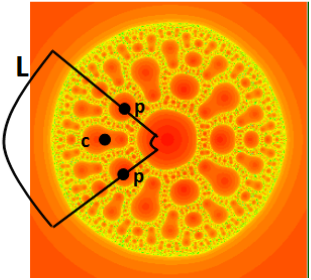

- Let L^λ be the closed portion of the wedge with inner boundary in the trapdoor, outer boundary in the basin, and straight line boundaries that are part of the two adjacent prepole rays as shown.

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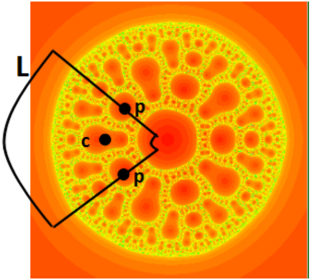
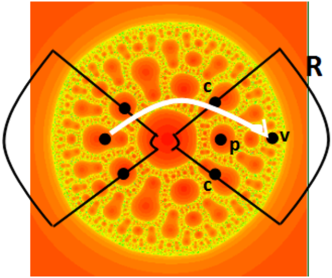


- Let L^λ be the closed portion of the wedge with inner boundary in the trapdoor, outer boundary in the basin, and straight line boundaries that are part of the two adjacent prepole rays as shown.
- There is one critical point c_0^λ in the interior of L^λ.

The right wedge R^λ



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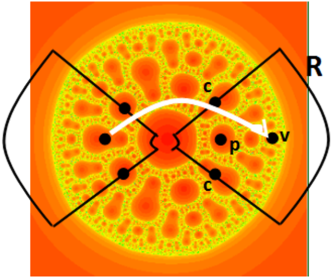

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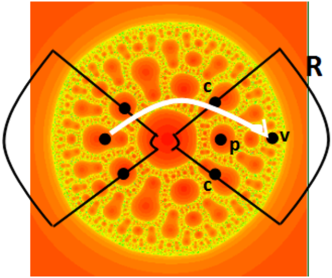

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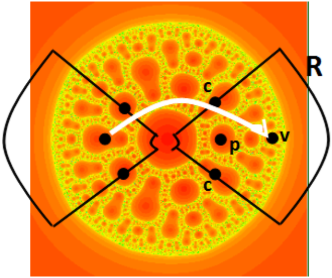
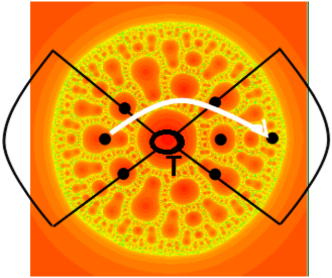


- Let R^λ be the symmetric right wedge. The straight line boundaries are part of two adjacent critical point rays.
- There is one prepole p_2^λ in the interior of R^λ.
- $v_0^\lambda = F_\lambda(c_0^\lambda)$ is in R^λ.

The (subset of the) trapdoor $T_{\mathcal{A}}$



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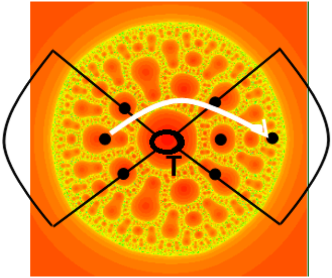

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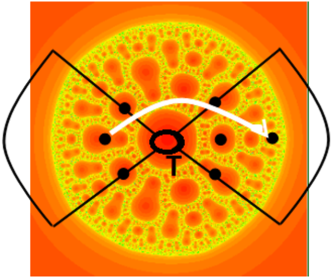

- Let $T_{\mathcal{A}}$ be the closed subset of the trapdoor containing 0 such that $L^{\lambda} \cup T_{\mathcal{A}} \cup R^{\lambda}$ are connected, and they only intersect along boundaries.
- This union will be referred to informally as the bowtie.

Proposition

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For each λ in that annular region:

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1. F_λ maps R^λ in 1-1 fashion onto a region that contains the interiors of $L^\lambda \cup T_{\mathcal{A}} \cup R^\lambda$;

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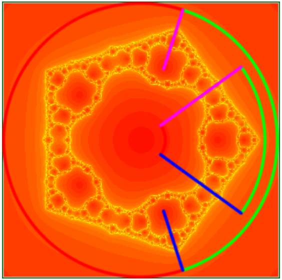
2. F_λ maps L^λ two-to-one over a region that contains the interior of R^λ;
3. Some stuff about the critical value winding around the boundary of R_0^λ that is not worth stating, justifying, or using for this talk.

Justification for part 1



- The critical point ray boundaries of R^{λ} are mapped two-to-one onto the critical value rays.

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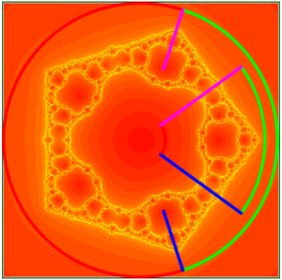

- The critical point ray boundaries of R^λ are mapped two-to-one onto the critical value rays. For each λ in the annular region, the critical value rays are disjoint from the interiors of L^λ, R^λ, and $T_{\mathcal{A}}$.

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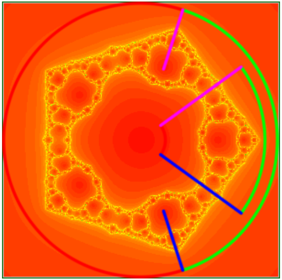

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- The boundary of R^λ in B^λ maps to the outer arc on the right.

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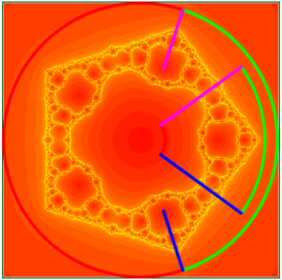

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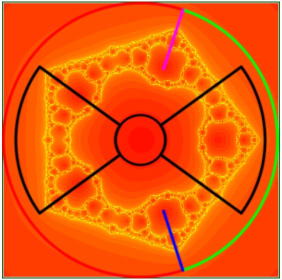

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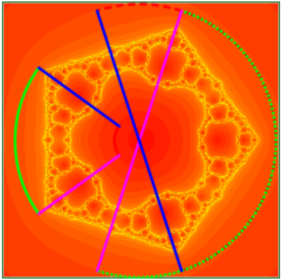
- The boundary of R^λ in B^λ maps to the outer arc on the right.
- The boundary of R^λ in $T_{\mathcal{A}}$ maps to the outer arc on the left.
- Then the image of R^λ properly contains the interiors R^λ, L^λ, and $T_{\mathcal{A}}$.

Justification for part 2



- The prepole rays map to the critical point rays passing through the origin.

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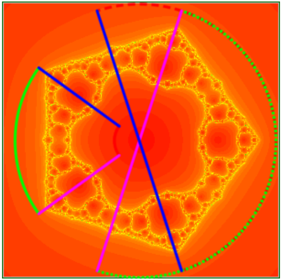

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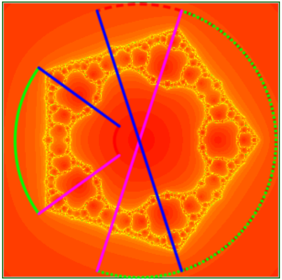

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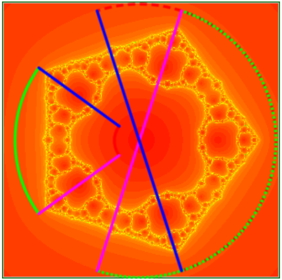

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- The boundary of L^λ in B^λ maps to an arc on the right.
- The boundary of L^λ in $T_{\mathcal{A}}$ maps to a slightly longer arc on the right.

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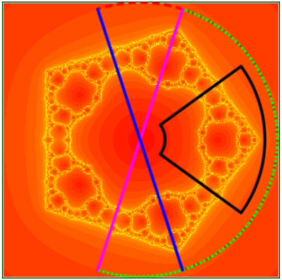

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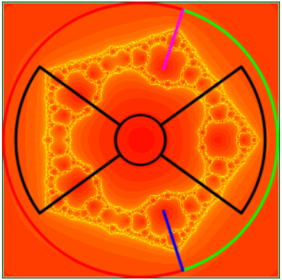
- The boundary of L^λ in B^λ maps to an arc on the right.
- The boundary of L^λ in $T_\mathcal{A}$ maps to a slightly longer arc on the right.
- Then F_λ maps L^λ over R^λ in two-to-one fashion.

Are the rays really disjoint for all λ?



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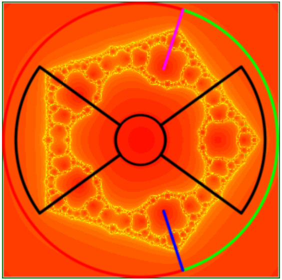

- Rotating λ clockwise or CCW by half a turn rotates the "bowtie" by one tenth of a turn.

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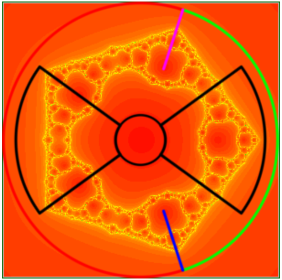
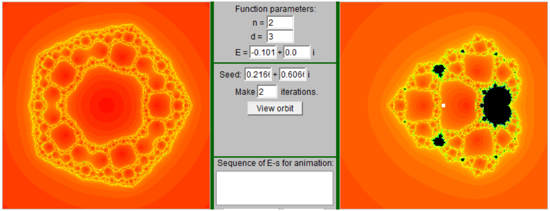


- Rotating λ clockwise or CCW by half a turn rotates the "bowtie" by one tenth of a turn. The critical value rays rotate one fifth of a turn but remain disjoint from R_0^λ.

Drawing a picture



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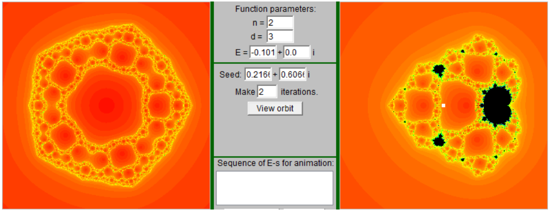
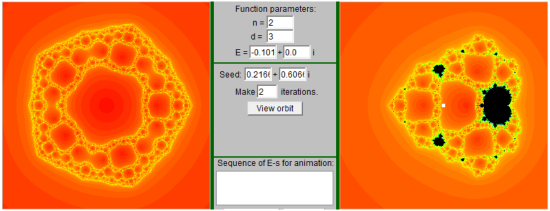


- We can "put a bowtie" on the dynamical plane, and the R^{λ} portion of the bowtie contains a preimage of the bowtie.

Drawing a picture



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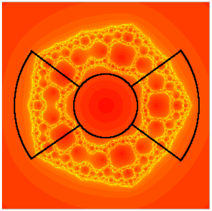
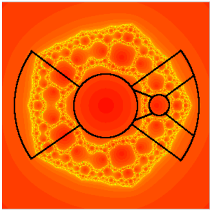


Preimage of the bowtie in the bowtie



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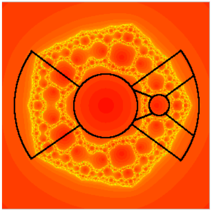

- Here is the preimage of the bowtie containing a preimage of R^λ.

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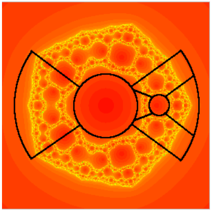
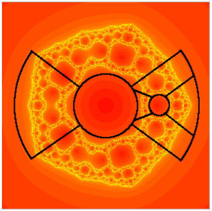


- Here is the preimage of the bowtie containing a preimage of R^λ.
- The preimage of R^λ in that preimage of the bowtie contains a preimage of the preimage of the bowtie.

Preimage of the bowtie in the bowtie



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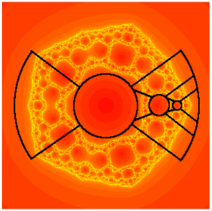
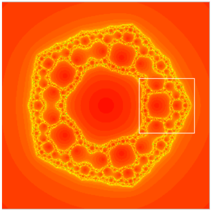


Zooming in



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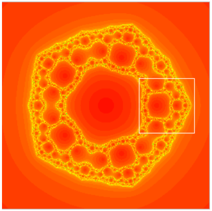

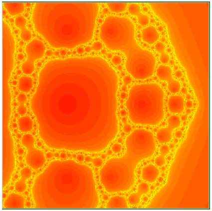

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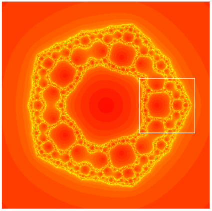

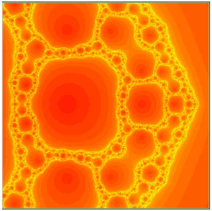

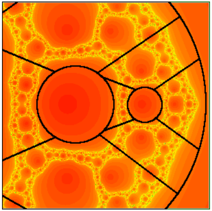

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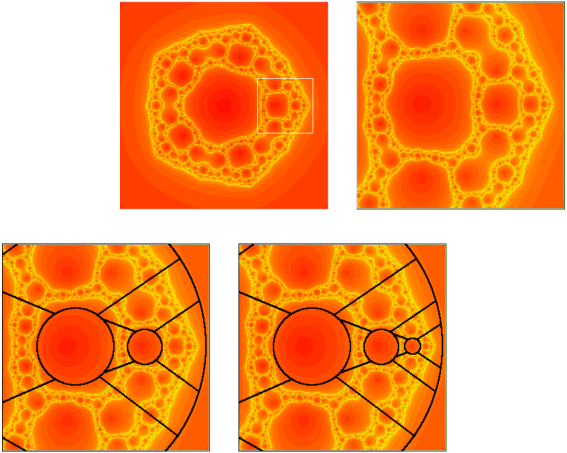

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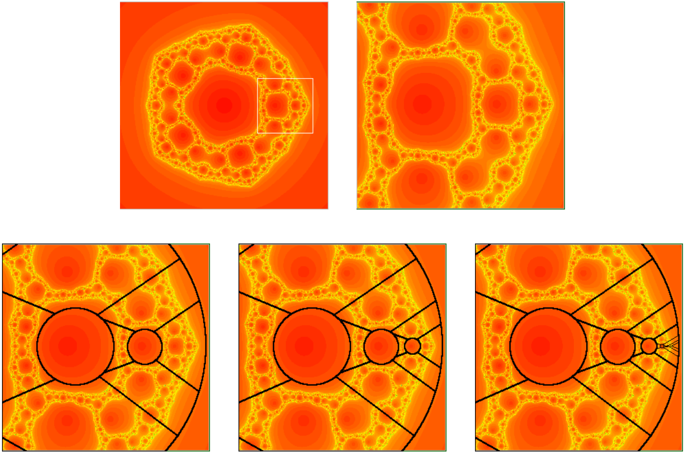
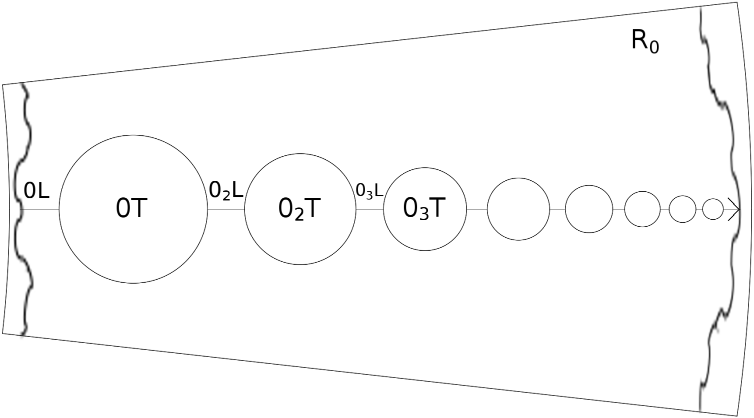
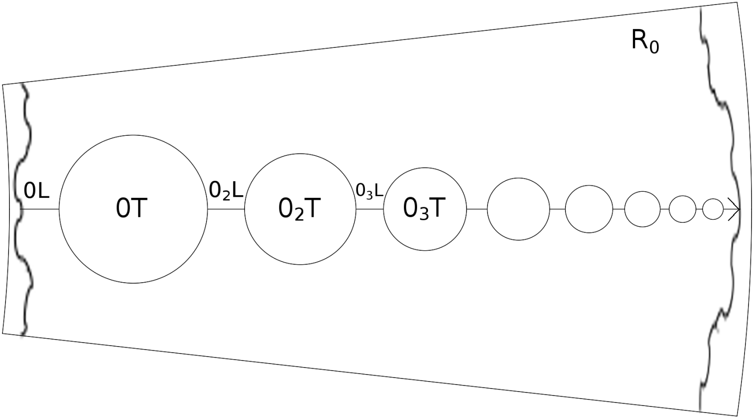


The dynamical $\bar{0}TL$ arc



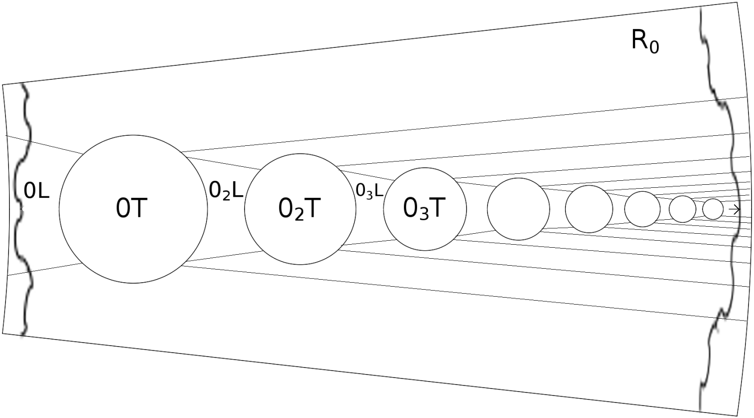
- Here is a stylized representation of the $\bar{0}TL$ arc in the dynamical plane.

The dynamical $\bar{0}TL$ arc



- Here is a stylized representation of the $\bar{0}TL$ arc in the dynamical plane. This is the arc of infinitely many preimages of L^λ and $T_{\mathcal{A}}$ in R^λ that accumulates at the fixed point in R^λ.

The dynamical $\bar{0}TL$ arc



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Claim

- The TL arc in the dynamical plane implies some corresponding structure exists in the parameter plane.

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- The *TL* arc in the dynamical plane implies some corresponding structure exists in the parameter plane.
- So how do these closed regions in the dynamical plane prove the existence of structures in the parameter plane?

$T_{\mathcal{A}} \implies$ S-hole

- Each preimage of $T_{\mathcal{A}}$ proves the existence of a Sierpinski hole on the negative real axis of the parameter plane.

$T_{\mathcal{A}} \implies$ S-hole

- Each preimage of $T_{\mathcal{A}}$ proves the existence of a Sierpinski hole on the negative real axis of the parameter plane.
- There is a prepole in R^{λ}.

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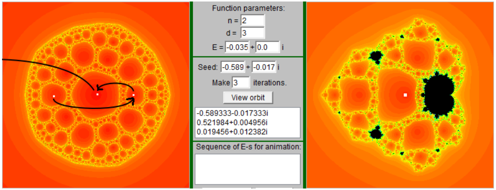

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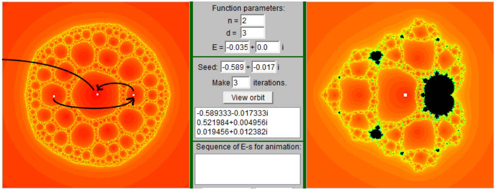

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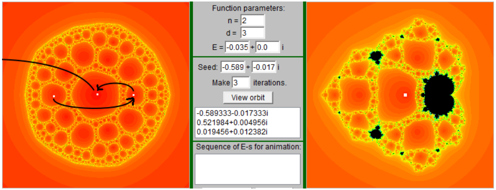

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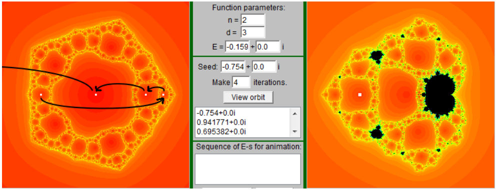

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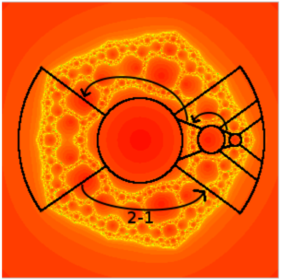

dynamical TL arc $\implies$ parameter SM arc

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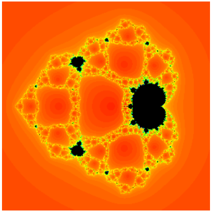


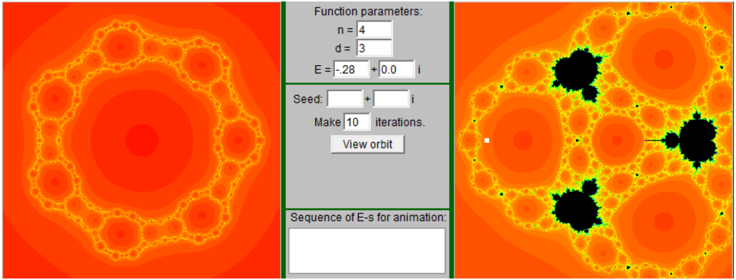
Outline

1. Introduction

2. $z^2 + \lambda/z^3$

3. $z^4 + \lambda/z^3$

The parameter and dynamical plane for $n = 4, d = 3$



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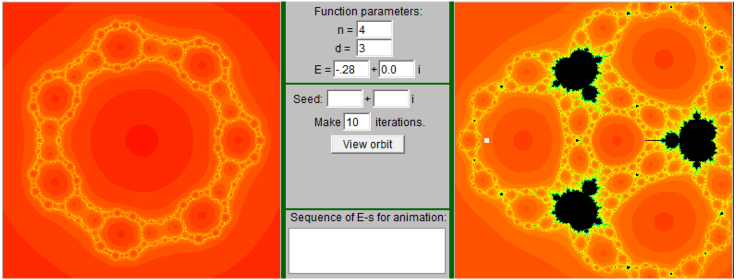

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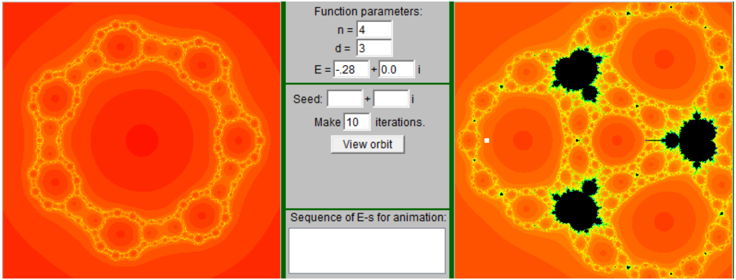

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- We need to check λ rotated one sixth of a turn CC and CW, instead of one half of a turn, which is great!

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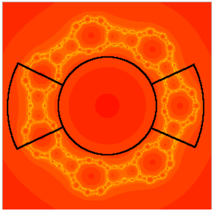

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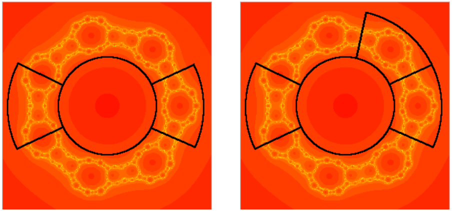

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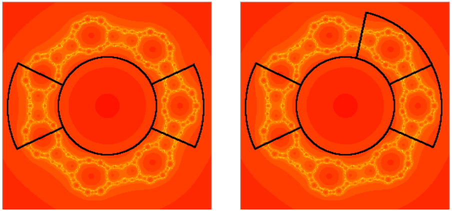

- We will refer to $L^\lambda \cup T_{\mathcal{A}} \cup R_0^\lambda \cup R_1^\lambda$ as the "lopsided bowtie."

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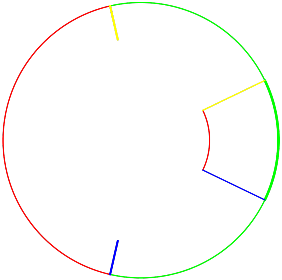
4. The winding index part again.

Analogous justification for part 1 (and part 2)



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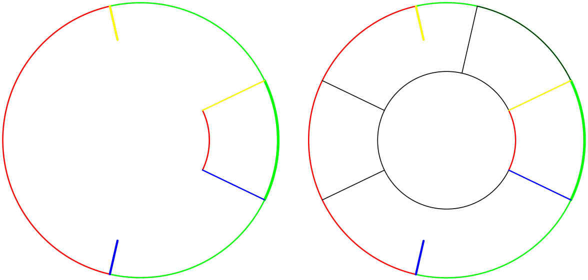

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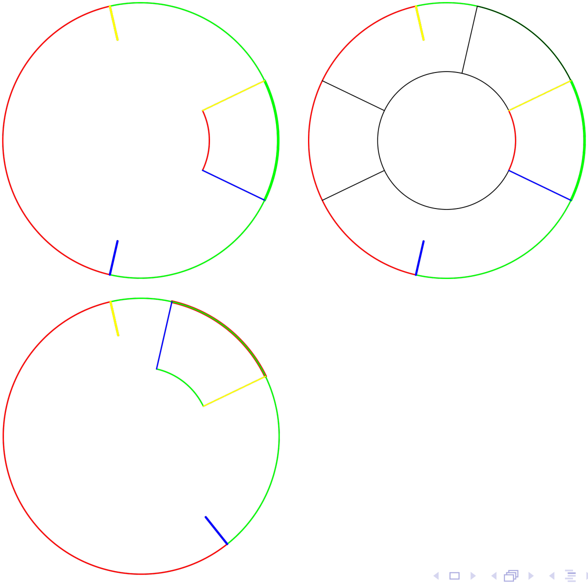

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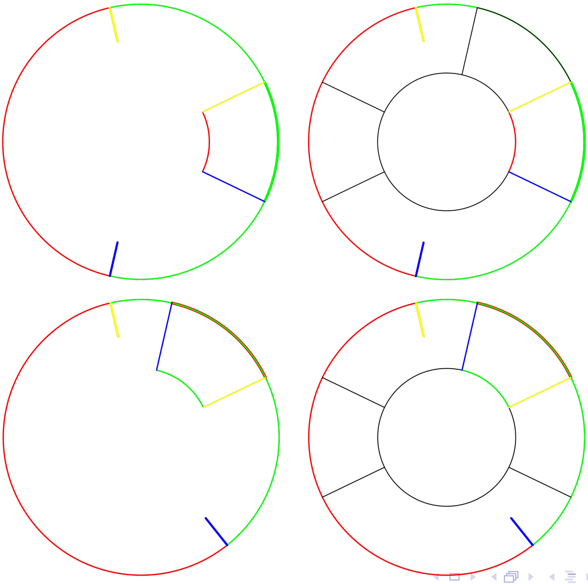
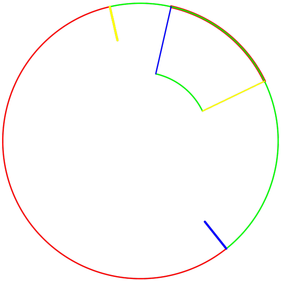


The bowtie in R_1^λ is rotated



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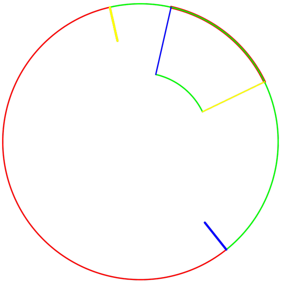

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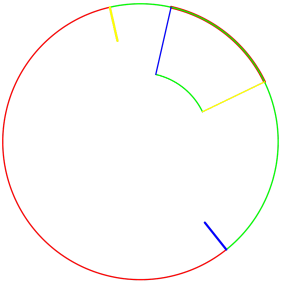

- The orientation is preserved, but note that the outer boundary of R_1^λ maps to the left side, while the inner boundary maps to the right. This means the bowtie is rotated inside R_1^λ (and all preimages of R_1^λ).

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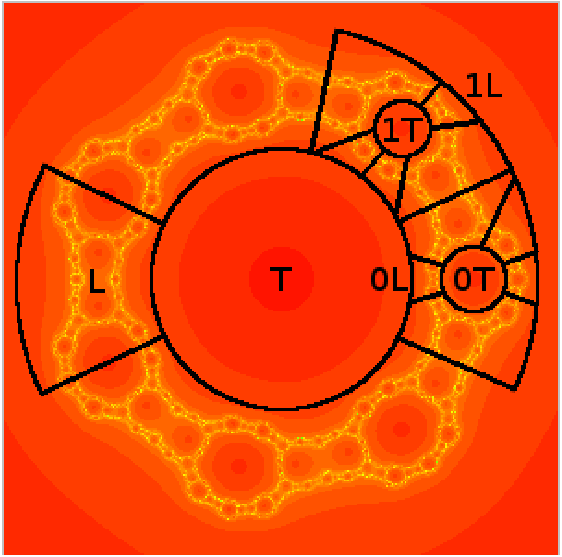
Labeling



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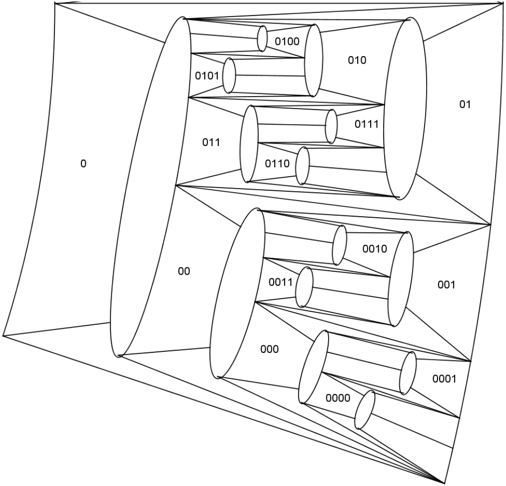

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- If we ignore R_1^λ and its preimages, we still have the $\overline{0}TL$ arc in the dynamical plane.
- It is named the $\overline{0}TL$ arc because one can think of z passing through the trap door, then $0L$, then $0T$, then 0_2L, then 0_2T, ... and accumulating at the fixed point in R_0^λ with the name $\overline{0}$.
- But the $\overline{0}TL$ arc exists entirely in R_0^λ. R_1^λ contains a preimage of R_0^λ. Therefore, R_1^λ contains a preimage of the $\overline{0}TL$ arc. The preimage of the $\overline{0}TL$ arc in R_1^λ is the $1\overline{0}TL$ arc.

Analogous TL arc

- If we ignore R_1^λ and its preimages, we still have the $\overline{0}TL$ arc in the dynamical plane.
- It is named the $\overline{0}TL$ arc because one can think of z passing through the trap door, then $0L$, then $0T$, then 0_2L, then 0_2T, ... and accumulating at the fixed point in R_0^λ with the name $\overline{0}$.
- But the $\overline{0}TL$ arc exists entirely in R_0^λ. R_1^λ contains a preimage of R_0^λ. Therefore, R_1^λ contains a preimage of the $\overline{0}TL$ arc. The preimage of the $\overline{0}TL$ arc in R_1^λ is the $1\overline{0}TL$ arc.
- (The preimage of the $\overline{0}TL$ arc in R_0^λ is itself.)

Infinitely many TL arcs

- Let's keep going:

Infinitely many TL arcs

- Let's keep going:
  The preimage of the $1\overline{0}TL$ arc in R_1^λ is the $1_2\overline{0}TL$ arc.

Infinitely many TL arcs

- Let's keep going:
  The preimage of the $1\overline{0}TL$ arc in R_1^λ is the $1_2\overline{0}TL$ arc.
  The preimage of the $1\overline{0}TL$ arc in R_0^λ is the $01\overline{0}TL$ arc.

Infinitely many TL arcs

- Let's keep going:
  The preimage of the $1\overline{0}TL$ arc in R_1^λ is the $1_2\overline{0}TL$ arc.
  The preimage of the $1\overline{0}TL$ arc in R_0^λ is the $01\overline{0}TL$ arc.
- The preimage of the $1_2\overline{0}TL$ arc in R_1^λ is the $1_3\overline{0}TL$ arc.

Infinitely many TL arcs

- Let's keep going:
  The preimage of the $1\overline{0}TL$ arc in R_1^λ is the $1_2\overline{0}TL$ arc.
  The preimage of the $1\overline{0}TL$ arc in R_0^λ is the $01\overline{0}TL$ arc.
- The preimage of the $1_2\overline{0}TL$ arc in R_1^λ is the $1_3\overline{0}TL$ arc.
  The preimage of the $1_2\overline{0}TL$ arc in R_0^λ is the $01_2\overline{0}TL$ arc.

Infinitely many TL arcs

- Let's keep going:
  The preimage of the $1\overline{0}TL$ arc in R_1^λ is the $1_2\overline{0}TL$ arc.
  The preimage of the $1\overline{0}TL$ arc in R_0^λ is the $01\overline{0}TL$ arc.
- The preimage of the $1_2\overline{0}TL$ arc in R_1^λ is the $1_3\overline{0}TL$ arc.
  The preimage of the $1_2\overline{0}TL$ arc in R_0^λ is the $01_2\overline{0}TL$ arc.
  The preimage of the $01\overline{0}TL$ arc in R_1^λ is the $101\overline{0}TL$ arc.

Infinitely many TL arcs

- Let's keep going:
  The preimage of the $1\bar{0}TL$ arc in R_1^λ is the $1_2\bar{0}TL$ arc.
  The preimage of the $1\bar{0}TL$ arc in R_0^λ is the $01\bar{0}TL$ arc.
- The preimage of the $1_2\bar{0}TL$ arc in R_1^λ is the $1_3\bar{0}TL$ arc.
  The preimage of the $1_2\bar{0}TL$ arc in R_0^λ is the $01_2\bar{0}TL$ arc.
  The preimage of the $01\bar{0}TL$ arc in R_1^λ is the $101\bar{0}TL$ arc.
  The preimage of the $01\bar{0}TL$ arc in R_0^λ is the $0_21\bar{0}TL$ arc.

Infinitely many TL arcs

- Let's keep going:
  The preimage of the $1\overline{0}TL$ arc in R_1^λ is the $1_2\overline{0}TL$ arc.
  The preimage of the $1\overline{0}TL$ arc in R_0^λ is the $01\overline{0}TL$ arc.
- The preimage of the $1_2\overline{0}TL$ arc in R_1^λ is the $1_3\overline{0}TL$ arc.
  The preimage of the $1_2\overline{0}TL$ arc in R_0^λ is the $01_2\overline{0}TL$ arc.
  The preimage of the $01\overline{0}TL$ arc in R_1^λ is the $101\overline{0}TL$ arc.
  The preimage of the $01\overline{0}TL$ arc in R_0^λ is the $0_21\overline{0}TL$ arc.
- That's probably enough.

Fixed points in Symbolic dynamics

- $\overline{0}$ is a fixed point in Σ_2.

Fixed points in Symbolic dynamics

- $\overline{0}$ is a fixed point in Σ_2. As is $\overline{1}$.

Fixed points in Symbolic dynamics

- $\overline{0}$ is a fixed point in Σ_2. As is $\overline{1}$. What would $\overline{1}$ mean in the context of our problem?

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- $\overline{0}$ is a fixed point in Σ_2. As is $\overline{1}$. What would $\overline{1}$ mean in the context of our problem?
- $\overline{0}$ is the fixed point of F_λ that lies in R_0^λ. $\overline{1}$ is the fixed point of F_λ that lies in R_1^λ.

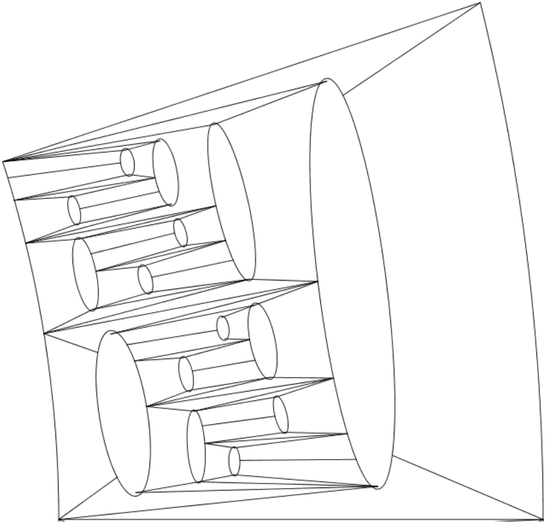
Fixed points in Symbolic dynamics

- $\overline{0}$ is a fixed point in Σ_2. As is $\overline{1}$. What would $\overline{1}$ mean in the context of our problem?
- $\overline{0}$ is the fixed point of F_λ that lies in R_0^λ. $\overline{1}$ is the fixed point of F_λ that lies in R_1^λ.
- The fixed point in R_0^λ lies on the boundary of the basin. The fixed point in R_1^λ does not...

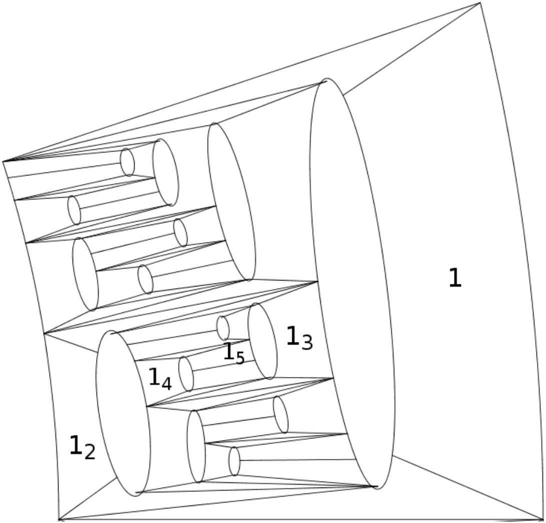
Fixed points in Symbolic dynamics

- $\overline{0}$ is a fixed point in Σ_2. As is $\overline{1}$. What would $\overline{1}$ mean in the context of our problem?
- $\overline{0}$ is the fixed point of F_λ that lies in R_0^λ. $\overline{1}$ is the fixed point of F_λ that lies in R_1^λ.
- The fixed point in R_0^λ lies on the boundary of the basin. The fixed point in R_1^λ does not...
- If we draw lopsided bowties in R_1^λ until we get tired of doing so, we get something like:

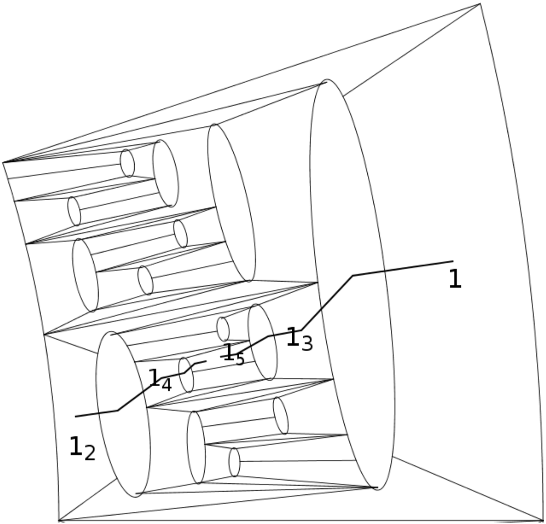
Stylized lopsided bowties



All $1's$ only



All $1's$ only



Claim

- There exists the $\overline{0}TL$ arc in R_0^λ for the rational map for $n = 4, d = 3$

Claim

- There exists the $\overline{0}TL$ arc in R_0^λ for the rational map for $n = 4, d = 3$ with the arc beginning on the boundary of T_λ

Claim

- There exists the $\overline{0}TL$ arc in R_0^λ for the rational map for $n = 4, d = 3$ with the arc beginning on the boundary of T_λ and accumulating at the fixed point on the boundary of B_λ.

Claim

- There exists the $\overline{0}TL$ arc in R_0^λ for the rational map for $n = 4, d = 3$ with the arc beginning on the boundary of T_λ and accumulating at the fixed point on the boundary of B_λ.
- There exists a different TL arc in R_1^λ for the rational map for $n = 4, d = 3$

Claim

- There exists the $\overline{0}TL$ arc in R_0^λ for the rational map for $n = 4, d = 3$ with the arc beginning on the boundary of T_λ and accumulating at the fixed point on the boundary of B_λ.
- There exists a different TL arc in R_1^λ for the rational map for $n = 4, d = 3$ such that the arc grows from both the boundary in T_λ and the boundary in B_λ,

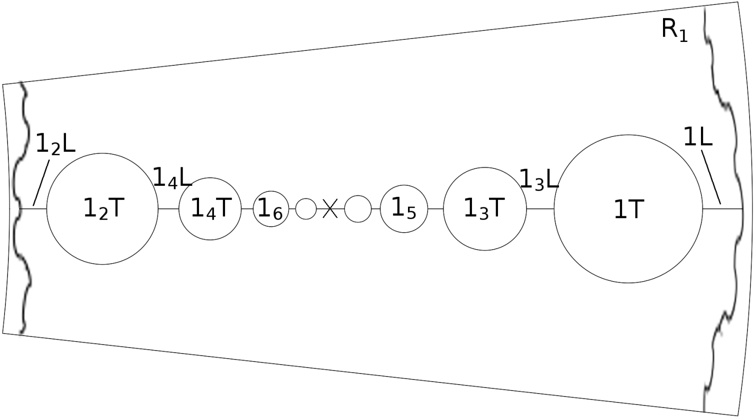
Claim

- There exists the $\overline{0}TL$ arc in R_0^λ for the rational map for $n = 4, d = 3$ with the arc beginning on the boundary of T_λ and accumulating at the fixed point on the boundary of B_λ.
- There exists a different TL arc in R_1^λ for the rational map for $n = 4, d = 3$ such that the arc grows from both the boundary in T_λ and the boundary in B_λ, and accumulates at the fixed point in the interior of R_1^λ.

Claim

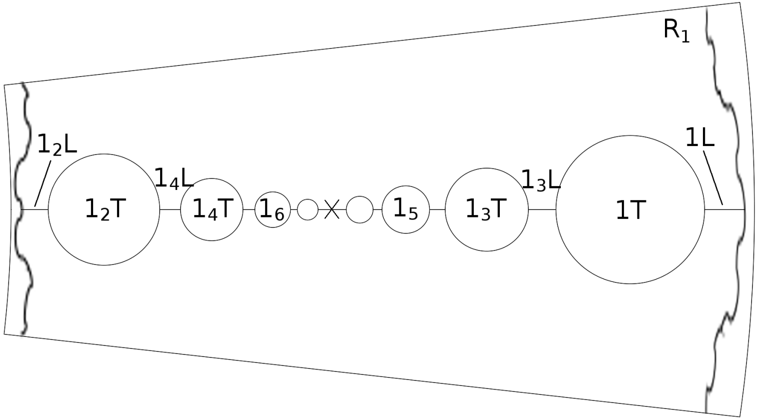
- There exists the $\overline{0}TL$ arc in R_0^λ for the rational map for $n = 4, d = 3$ with the arc beginning on the boundary of T_λ and accumulating at the fixed point on the boundary of B_λ.
- There exists a different TL arc in R_1^λ for the rational map for $n = 4, d = 3$ such that the arc grows from both the boundary in T_λ and the boundary in B_λ, and accumulates at the fixed point in the interior of R_1^λ.
- The proof is basically looking at the rotated lopsided bowties inside each preimage of R_1^λ.

The dynamical $\bar{1}TL$ arc



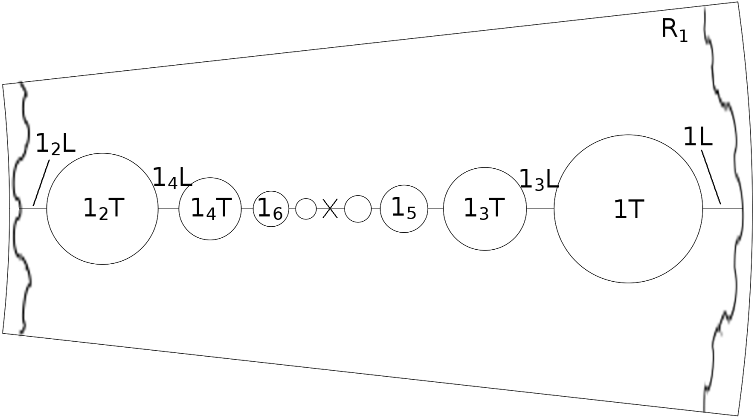
- Here is a stylized representation of the $\bar{1}TL$ arc in the dynamical plane.

The dynamical $\overline{1}TL$ arc



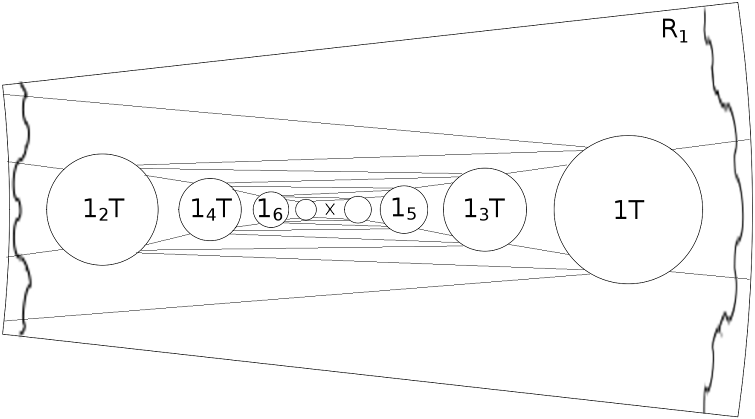
- Here is a stylized representation of the $\overline{1}TL$ arc in the dynamical plane. This is the arc of infinitely many preimages of L^λ and $T_\mathcal{A}$ in R_1^λ that accumulates at the fixed point in R_1^λ.

The dynamical $\bar{1}TL$ arc



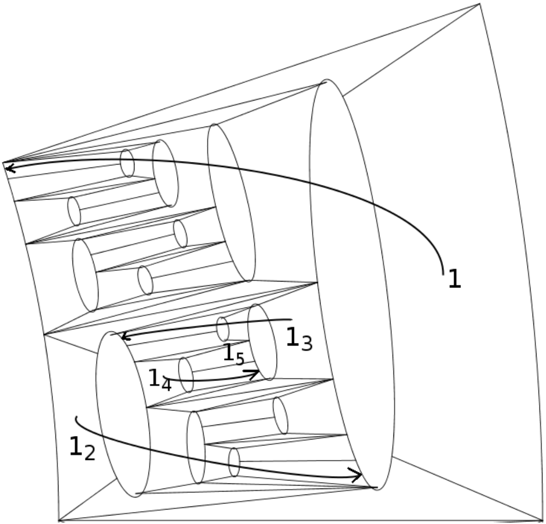
- Here is a stylized representation of the $\bar{1}TL$ arc in the dynamical plane. This is the arc of infinitely many preimages of L^λ and $T_{\mathcal{A}}$ in R_1^λ that accumulates at the fixed point in R_1^λ. Every preimage of L^λ and $T_{\mathcal{A}}$ with orbit not including R_0^λ is part of the $\bar{1}TL$ arc.

The dynamical $\bar{1}TL$ arc

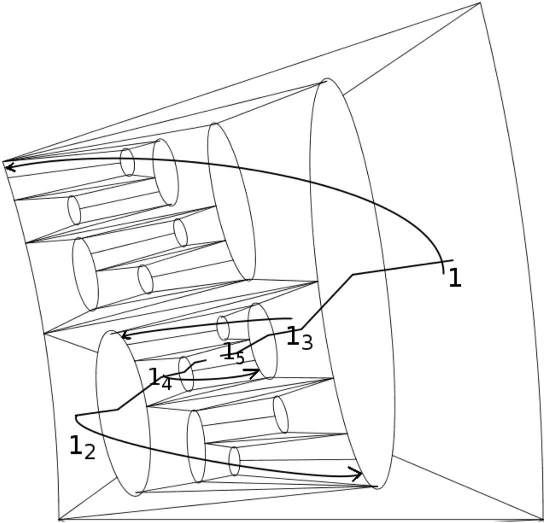


- Here is a stylized representation of the $\bar{1}TL$ arc in the dynamical plane. This is the arc of infinitely many preimages of L^λ and $T_{\mathcal{A}}$ in R_1^λ that accumulates at the fixed point in R_1^λ. Every preimage of L^λ and $T_{\mathcal{A}}$ with orbit not including R_0^λ is part of the $\bar{1}TL$ arc.

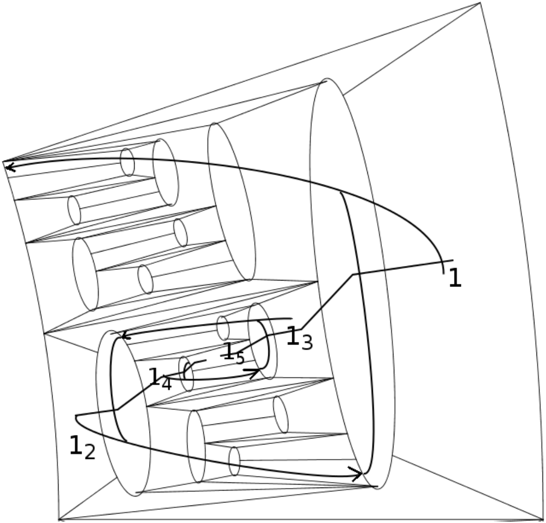
Infinitely many $\bar{0}TL$ arcs



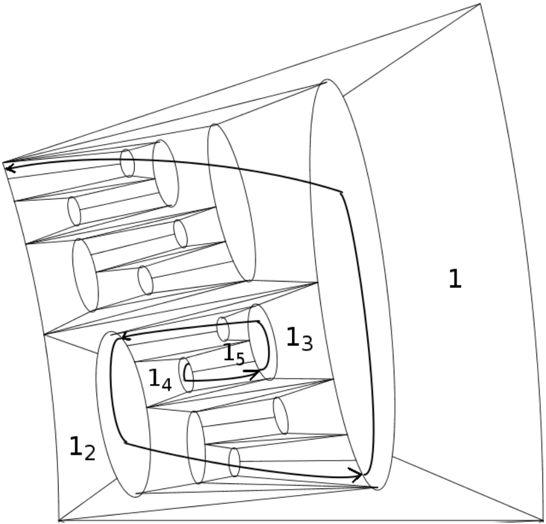
Infinitely many $\bar{0}TL$ arcs intersecting the $\bar{1}TL$ arc



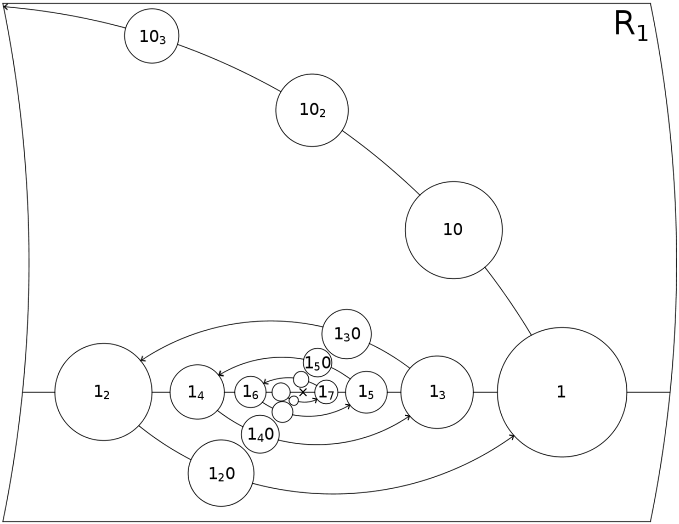
A continuous path for λ



The $\overline{1}TL$ spiral



Another representation of the $\bar{1}TL$ spiral



Claim

- There is an arc of infinitely many alternating preimages of L^λ and $T_\mathcal{A}$ in R_1^λ in the dynamical plane.

Claim

- There is an arc of infinitely many alternating preimages of L^λ and $T_\mathcal{A}$ in R_1^λ in the dynamical plane.
- This dynamical arc does NOT prove the existence of infinitely many alternating Sierpinski holes and Mandelbrot sets in the parameter plane.

Claim

- There is an arc of infinitely many alternating preimages of L^λ and $T_{\mathcal{A}}$ in R_1^λ in the dynamical plane.
- This dynamical arc does NOT prove the existence of infinitely many alternating Sierpinski holes and Mandelbrot sets in the parameter plane.
- Recall that F_λ maps L^λ two-to-one over R_0^λ, which contains the lopsided bowtie.

Claim

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- Recall that F_λ maps L^λ two-to-one over R_0^λ, which contains the lopsided bowtie. The statement is NOT true if we replace R_0^λ with R_1^λ.

Claim

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- Recall that F_λ maps L^λ two-to-one over R_0^λ, which contains the lopsided bowtie. The statement is NOT true if we replace R_0^λ with R_1^λ.
- However, R_0^λ contains a preimage of R_1^λ.

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- Recall that F_λ maps L^λ two-to-one over R_0^λ, which contains the lopsided bowtie. The statement is NOT true if we replace R_0^λ with R_1^λ.
- However, R_0^λ contains a preimage of R_1^λ. Then the preimage of the $\overline{1}TL$ arc, the $0\overline{1}TL$ arc, is in R_0^λ.

Claim

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- However, R_0^λ contains a preimage of R_1^λ. Then the preimage of the $\overline{1}TL$ arc, the $0\overline{1}TL$ arc, is in R_0^λ.
- This dynamical arc proves the existence of an SM arc in the parameter plane.

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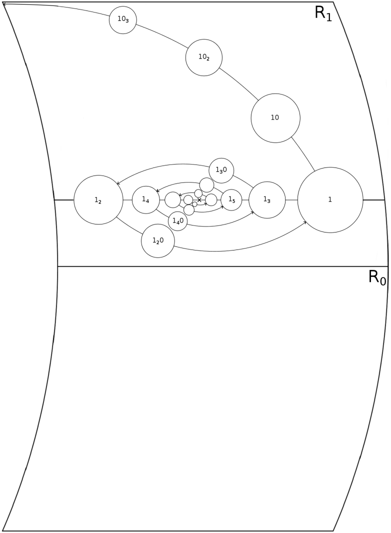
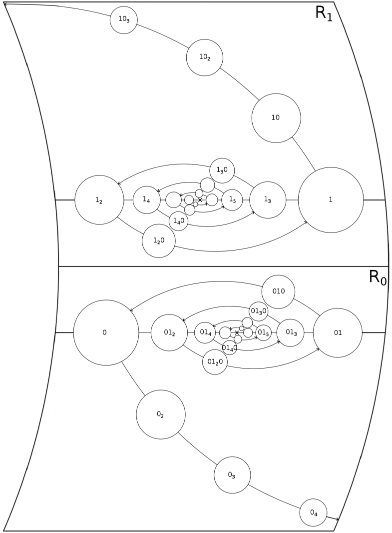
- Recall that F_λ maps L^λ two-to-one over R_0^λ, which contains the lopsided bowtie. The statement is NOT true if we replace R_0^λ with R_1^λ.
- However, R_0^λ contains a preimage of R_1^λ. Then the preimage of the $\overline{1}TL$ arc, the $0\overline{1}TL$ arc, is in R_0^λ.
- This dynamical arc proves the existence of an SM arc in the parameter plane.
- And infinitely many $\overline{0}$ SM arcs pass through the $0\overline{1}$ SM arc to make the $0\overline{1}$ SM spiral.

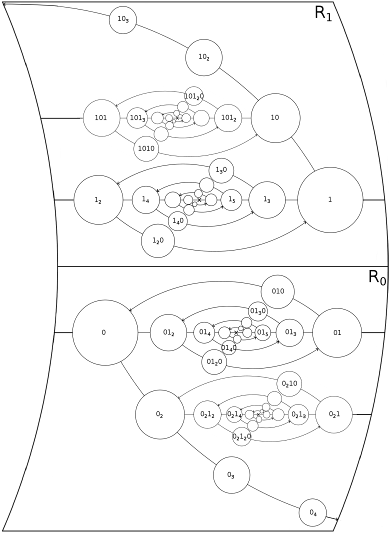
Infinitely many *TL* spirals



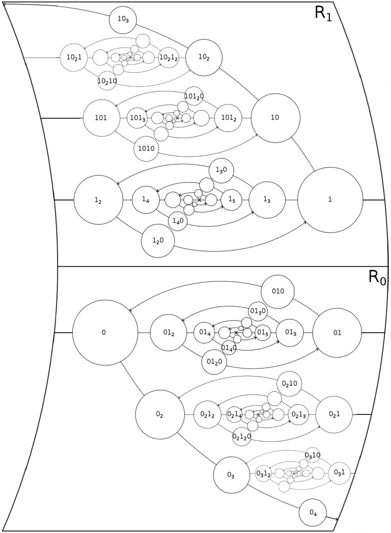
Infinitely many *TL* spirals



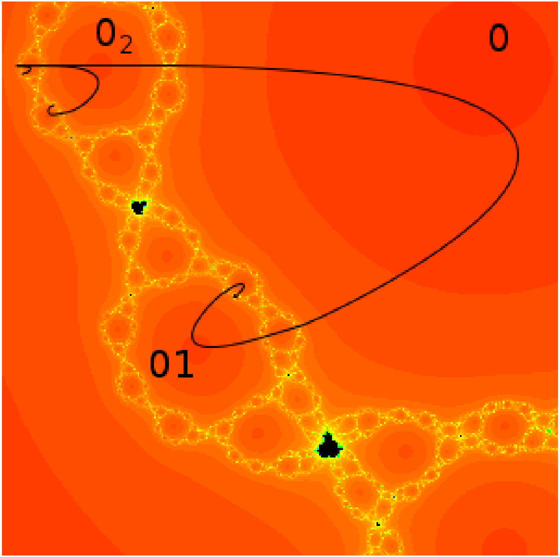
Infinitely many *TL* spirals



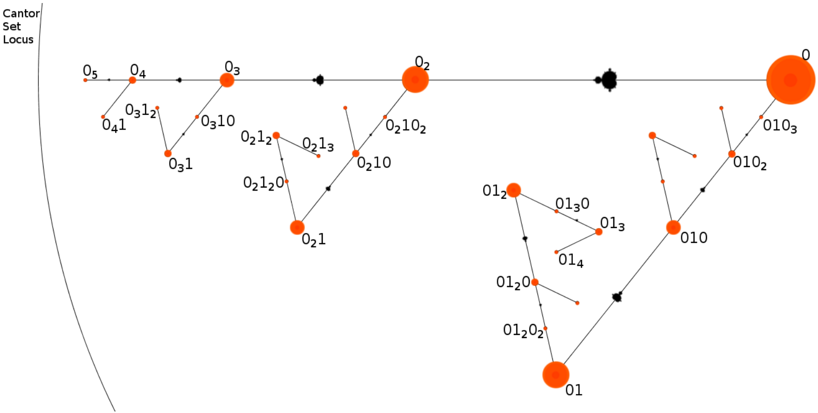
Infinitely many *TL* spirals



Infinitely many SM spirals



Infinitely many stylized SM spirals



How many times I can use the word infinitely?

- We have found the $\overline{0}TL$ arcs of infinitely many Sierpinski holes and Mandelbrot sets, and infinitely many of its preimages.

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- We have found the $\overline{0}TL$ arcs of infinitely many Sierpinski holes and Mandelbrot sets, and infinitely many of its preimages.
- We have also found a different $\overline{1}TL$ arc of infinitely many Sierpinski holes and Mandelbrot sets.

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- Infinitely many $\overline{0}TL$ arcs pass through a $\overline{1}TL$ arc to make a $\overline{1}TL$ spiral.

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How many times I can use the word infinitely?

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- Infinitely many $\overline{0}TL$ arcs pass through a $\overline{1}TL$ arc to make a $\overline{1}TL$ spiral. There are infinitely many preimages of this spiral.
- None of the infinitely many dynamical structures in R_1^λ can be used to prove the existence of structures in the parameter plane, but there are infinitely many preimages of these structures in R_0^λ.

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- None of the infinitely many dynamical structures in R_1^λ can be used to prove the existence of structures in the parameter plane, but there are infinitely many preimages of these structures in R_0^λ.
- Then the $\overline{0}$ SM arc and infinitely many of its "preimages" exist in the parameter plane.

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- Infinitely many $\overline{0}TL$ arcs pass through a $\overline{1}TL$ arc to make a $\overline{1}TL$ spiral. There are infinitely many preimages of this spiral.
- None of the infinitely many dynamical structures in R_1^λ can be used to prove the existence of structures in the parameter plane, but there are infinitely many preimages of these structures in R_0^λ.
- Then the $\overline{0}$ SM arc and infinitely many of its "preimages" exist in the parameter plane.

  The $0\overline{1}$ SM arc and infinitely many of its "preimages" exist in the parameter plane.

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- Infinitely many $\overline{0}TL$ arcs pass through a $\overline{1}TL$ arc to make a $\overline{1}TL$ spiral. There are infinitely many preimages of this spiral.
- None of the infinitely many dynamical structures in R_1^λ can be used to prove the existence of structures in the parameter plane, but there are infinitely many preimages of these structures in R_0^λ.
- Then the $\overline{0}$ SM arc and infinitely many of its "preimages" exist in the parameter plane.
  The $0\overline{1}$ SM arc and infinitely many of its "preimages" exist in the parameter plane.
  The $0\overline{1}$ SM spiral and infinitely many of its "preimages" exist in the parameter plane.

Thank you!

Thank you for listening!

Outline

4 Insurance

Equivalent definitions of the Julia Set

$\mathcal{J}(F)$ is the set of all points at which the family of iterates of F fails to be a normal family in the sense of Montel.

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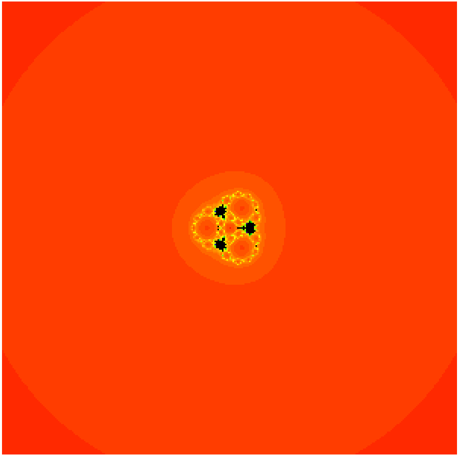
Equivalent definitions of the Julia Set

$\mathcal{J}(F)$ is the set of all points at which the family of iterates of F fails to be a normal family in the sense of Montel. Equivalently, $\mathcal{J}(F)$ is the closure of the set of repelling periodic points of F, and it is also the boundary of the set of points whose orbits tend to ∞ under iteration of F.

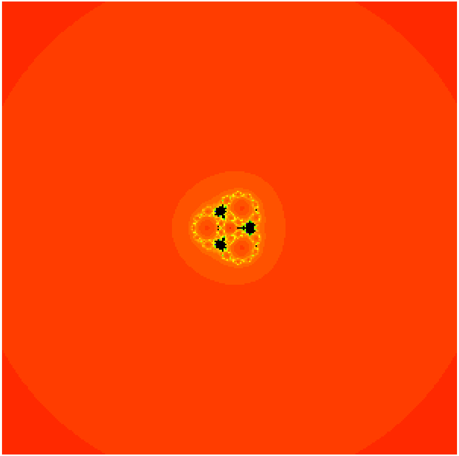
Equivalent definitions of the Julia Set

$\mathcal{J}(F)$ is the set of all points at which the family of iterates of F fails to be a normal family in the sense of Montel. Equivalently, $\mathcal{J}(F)$ is the closure of the set of repelling periodic points of F, and it is also the boundary of the set of points whose orbits tend to ∞ under iteration of F. back

λ restricted to an annular region



λ restricted to an annular region



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