The pseudo-arc

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Topics in Complex Dynamics

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Motivation:
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Recently Rempe-Gillen has characterised the topology of Julia continua of disjoint type functions. More precisely, he gave an almost complete description of the possible topology of their components.
There exists a disjoint-type entire function such that, for every connected component C of $J(f)$, the set $C \cup \{\infty\}$ is a *pseudo-arc*.

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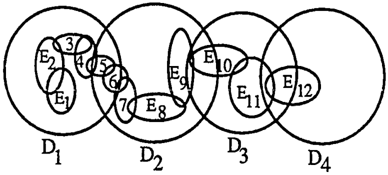

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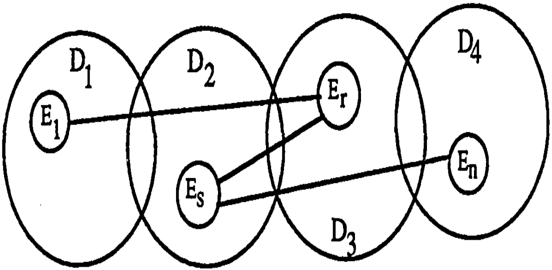

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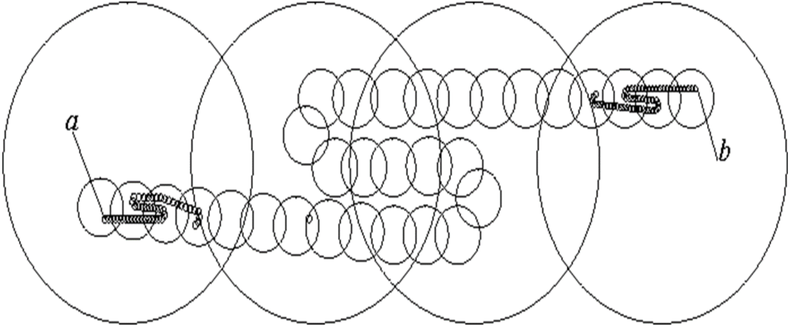
is called a *pseudo-arc*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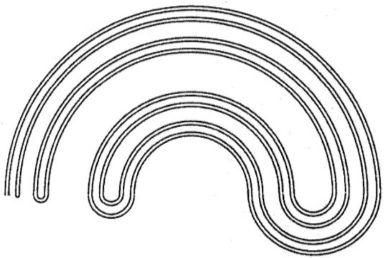

Figure : Knaster continuum

Theorem 1

A pseudo-arc is hereditarily indecomposable.

Thank you for your attention!