A TRANSCENDENTAL JULIA SET OF HAUSDORFF DIMENSION 1

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The study of the Hausdorff dimension of Julia sets of rational maps and transcendental (non-polynomial entire) functions are in some sense "opposite" of each other. In the rational case, it is easy to construct examples of Julia sets with Hausdorff dimension 1, while it takes more work to construct examples with Hausdorff dimension equal or close 2. It is much more difficult to reduce the Hausdorff dimension from 2 in the transcendental case. For example, two of the most natural familes of transcendental functions to consider, the exponential family ($f_\lambda(z) = \lambda e^z$) and sine family ($f_{a,b}(z) = \sin(az + b)$), always have Julia sets with Hausdorff dimension 2, see [3].

In 1975, Baker [1] proved that the Julia set must always have Hausdorff dimension ≥ 1 in the transcendental case, and Stallard [4] constructed examples where the Hausdorff dimension was arbitrarily close to 1. Recently, Bishop [2] constructed the first example of a transcendental Julia set of dimension 1, completing the proof that all Hausdorff dimensions in $[1, 2]$ are attained. In this talk, we will go over the construction of this function, discuss why its Julia set has dimension 1, and describe some of its other interesting dynamical properties.

REFERENCES

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