CONSTRUCTING AN ENTIRE FUNCTION WITH JULIA SET OF ZERO LEBESGUE MEASURE

WEIWEI CUI, CHRISTIAN-ALBRECHTS-UNIVERSITÄT ZU KIEL

We give conditions under which the Julia set and the escaping set of an entire function in the Eremenko-Lyubich class \mathcal{B} have zero Lebesgue measure. The essential ingredient in the proof is to show the model function in logarithmic coordinates has the corresponding property. Moreover, we construct an entire function of order of growth 1/2 in class \mathcal{B} . The escaping set of such an entire function has zero Lebesgue measure. To show it is in class \mathcal{B} we show first it belongs to the Laguerre-Pólya class and then use the Ahlfors distortion theorem. That the escaping set has zero Lebesgue measure follows easily from our first result. This would complement a result by Aspenberg and Bergweiler, and show the sharpness of their condition.

References

[AB12] M. Aspenberg and W. Bergweiler (2012) Entire functions with Julia sets of positive measure. Math. Ann., 352(1), 27-54.