TONGUES AND BIFURCATIONS ON A FAMILY OF DEGREE 4 BLASCHKE PRODUCTS

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We study the family of degree 4 Blaschke products $B_a(z) = z^3(z-a)/(1-\bar{a}z)$. It is the rational analogous of the double standard family of entire transcendental maps of the punctured plane given by $h(z) = e^{i\alpha}z^2e^{\beta/2(z-1/z)}$. Indeed, both families leave the unit circle $\mathbb{S}^1$ invariant and restrict, for certain parameters, to degree 2 coverings of $\mathbb{S}^1$. Even if the lift of $B_a|_{\mathbb{S}^1}$ is more complicated than in the transcendental case, the global dynamics of B_a are simpler (c.f. [1]). The non-holomorphic parametrization of the family B_a leads to the existence of phenomena on the parameter plane which cannot occur otherwise. For $|a| \geq 2$, there appear tongues as sets of parameters a for which B_a has an attracting cycles in the unit circle. They were studied by M. Misiurewicz and A. Rodrigues [2] for the family of double standard maps and are a degree 2 analogous of the Arnold Tongues.

During the talk we will present the main properties of the Blaschke products B_a. Afterwards we will study the tongues in the parameter plane of the Blaschke family, focusing on how bifurcations take place in a neighbourhood of their tips and how they may be extended beyond their natural range of definition.

References

[1] J. Canela, N. Fagella and A. Garijo. *On a family of rational perturbations of the doubling map.* J. Difference Equ. Appl. **21**(8), 715–741, 2015.
[2] M. Misiurewicz and A. Rodrigues. *Double standard maps.* Comm. Math. Phys. **273**(1), 37–65, 2007.