SUMMER SCHOOL: TOPICS IN COMPLEX DYNAMICS 2013

IMUB-UB, Barcelona. 10/6/2013 - 14/6/2013

SCHEDULE

	Monday (*)	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
9.00-10.30	K. Barański	J. H. Hubbard	Tan Lei	Tan Lei	K. Barański
10.30-11.00	*Coffee*	*Coffee*	*Coffee*	*Coffee*	*Coffee*
11.00-12.30	S. Godillon / S. Albrecht	K. Barański	J. H. Hubbard	J. H. Hubbard	Tan Lei
13.00-14.30	*Lunch*	*Lunch*	*Lunch*	*Lunch*	*Lunch*
15.30-16.00 16.00-16.30	R. Lodge S. Mukherjee	T. Gauthier M. Baumgartner		J. Tomasini C. Pérez	
16.30-17.00	*Coffee*	*Coffee*		*Coffee*	
17.00-17.30 17.30-18.00	T. Nayak K. Mamayusupov	N. A. Alhabib D. Martí		S. Vogel J. Canela	

(*) On Monday we will have the registration from 9 to 9.30. So all schedule moves half an hour down. Hence, S. Godillon and S. Albrecht's talks will be from 11.30 to 12.00 and 12.00 to 12.30, respectively.

MORNING COURSES

THERMODYNAMIC FORMALISM FOR TRANSCENDENTAL MAPS

Krzystof Barański (Warsaw University)

The course will present an introduction to the theory of thermodynamic formalism and its applications in the study of dynamics of entire and meromorphic transcendental maps.

Talk one. In the first talk, we will start with a description of the classical theory, developed by Ruelle, Bowen and Walters in the 1970's, with the emphasis on the notion of topological pressure and Bowen's formula for conformal repellers, in particular hyperbolic Julia sets of rational maps.

Talk two and three. During the second and third talk, we will describe some recent results which aim at generalization of the theory to the case of transcendental maps. These include works on the exponential family, finite order maps with balanced derivative grow and transcendental maps from class $\mathcal{S}$ (the set of singularities of the inverse map is finite) and class $\mathcal{B}$ (the set of singularities of the inverse map is bounded), due to Mayer, Urbański, Karpińska, Zdunik and the author.

DYNAMICS OF SEVERAL VARIABLES. STRUCTURE OF HÉNON MAPPINGS

John H. Hubbard (Cornell University / Université Marseille)

Talk one. Structure of Hénon mappings

Talk two. Parameter space for Hénon mappings

Talk three. Hilbert modular varieties

ENTROPY ON HUBBARD TREES

Tan Lei (Université d'Angers)

Talk one. We will present the theory of Milnor-Thurston (1977) on kneading determinants and semi-conjugacies to tent maps of unimodal maps, as well as the continuity and monotonicity property of the topological entropy of the quadratic family. This part concerns only real dynamical systems and is fairly elementary.

Talk two. We will present Douady's proof of the monotonicity of the topological entropy, using holomorphic dynamical tools such as dynamical and parametrical external rays.

Talk three. We will present several extensions of the above results to Hubbard trees of polynomials. These include: Alseda and Fagella's computations, Thurston's torus model of the abstract Hubbard tree and Tiozzo's work on veins of the Mandelbrot set.

LIST OF SHORT TALKS

On the construction of entire functions in the Speiser class

Simon Albrecht (Kiel University)

We construct entire functions with finite set of singular values and prescribed tracts by using quasiconformal folding, a method introduced by C. Bishop in 2011.

Explosion points for exponential maps

Nada A. Alhabib (Liverpool University)

Consider the family of exponential maps,

$$f_a(z) = e^z + a,\ a \in \mathbb{C}.$$

For certain parameters, including $a \in (-\infty, -1)$ the Julia set $\mathcal{J}(f_a)$ of this family is well understood. In fact, it is an uncountable union of curves, each consisting of a finite endpoint and a ray that connects this endpoint to infinity. John Mayer showed that (for these values of the parameter a), the set of endpoints $\mathcal{E}(f_a)$ has the surprising property that it is totally disconnected but $\mathcal{E}(f_a) \cup \{\infty\}$ is connected. In this talk, we shall prove that, for any exponential map $f_a(z)$, the set $\mathcal{E}^-(f_a)$ of *escaping endpoints* (as introduced by Schleicher and Zimmer) has the property that $\mathcal{E}^-(f_a) \cup \{\infty\}$ is connected. This is a joint work with my supervisor, Lasse Rempe-Gillen.

On boundaries of multiply connected wandering domains

Markus Baumgartner (Kiel University)

We show that under certain conditions the boundary of a multiply connected wandering domain is a countable number of Jordan curves. For example these conditions are satisfied by a family of functions that was first considered by Bergweiler and Zheng in 2011 and includes the first example of a multiply connected wandering domain by Baker.

Arnold tongues in degree 4 Blaschke products

Jordi Canela (Universitat de Barcelona)

The family of Blashcke products $B_a(z) = z^3(z-a)/(1-\bar{a}z)$ is the rational analogue of the double standard family given by $h(z) = e^{i\alpha}z^2e^{\beta/2(z-1/z)}$. Both families restrict, for certain parameters,

to degree 2 coverings of the unit circle. This fact leads to some interesting phenomena like the existence of tongues in the parameter plane. These tongues were studied for the first time by M. Misiurewicz and A. Rodrigues (2007) and are a degree 2 analogous of the Arnold Tongues.

During the talk we will introduce the concept of tongue for the Blashcke family and we will study what occurs around the tongues. We will also see how bifurcations take place along curves, in a similar way to what happens for antipolynomials $P_c(z) = \bar{z}^d + c$.

Distribution of postcritically finite polynomials

Thomas Gauthier (Université de Picardie Jules Verne)

This is a joint work with Charles Favre. We prove that Misiurewicz parameters with prescribed combinatorics and hyperbolic parameters with $(d-1)$ distinct attracting cycles with given multipliers are equidistributed with respect to the bifurcation measure in the moduli space of degree d complex polynomials. Our proof relies on Yuan's equidistribution results of points of small heights, and uses in a crucial way Epstein's transversality results.

From quasiconformal foldings to entire functions

Sébastien Godillon (Universitat de Barcelona)

I will introduce recent Bishop's works that provides a method using quasiconformal foldings to construct transcendental entire maps from certain infinite graphs embedded in the plane. We will show that these graphs contain some dynamical informations about the resulting maps. For instance, in the case of only two critical values and no asymptotic value, the graph is actually an infinite tree which corresponds to the preimage of the straight segment between the two critical values.

A combinatorial characterization of postcritically finite Newton maps

Russell Lodge (Jacobs University at Bremen)

It will be shown that every postcritically finite Newton map gives rise to a finite graph containing all postcritical points called an extended Newton graph (together with a graph map). On the other hand, an extended Newton graph can be extended to a branch covering from the two-sphere to itself, and using Thurston's theorem it will be shown that this branch covering is equivalent to a Newton map. This is joint work with Y. Mikulich and D. Schleicher.

Parabolic surgery and its application to Newton's method

Khudoyor Mamayusupov (Jacobs University at Bremen)

I will present surgery method on turning hyperbolics to parabolics done by Haissinky with sketch of proof and apply it to Newton maps.

The escaping set of entire functions

David Martí (Universitat de Barcelona / The Open University)

Given an entire function f, the escaping set I(f) consists of all the points whose orbit tends to infinity under iteration by f. It has been shown that I(f) is very related with the Julia set of f. In general this set has a complicated structure: it may be a disjoint union of uncountably many curves to infinity (Cantor bouquet), a parabolic domain at infinity (Baker domain), a wandering domain, etc. I will mention some open problems and try to summarize the main results about I(f), including the ones concerning slow and fast escaping points. I will focus on the notion of annular itineraries introduced recently by Phil Rippon and Gwyneth Stallard.

Combinatorics and topology of the Multicorns

Sabyasachi Mukherjee (Jacobs University at Bremen)

We consider the connectedness loci of degree d unicritical antipolynomials $f(z) = \bar{z}^d + c$, known as the Multicorns. The iteration theory of anti-polynomials differs only in a few subtle ways from that of ordinary polynomials. However, the parameter spaces of anti-polynomials are quite different from their holomorphic counterparts. After stating a few known results, we will describe the properties of anti-holomorphic orbit portraits, wake structures and behaviour of periodic parameter rays. Using these combinatorial data and the bifurcation phenomena, we deduce the number of hyperbolic components of a given period (which in general differs from the holomorphic case). In contrast to the holomorphic case, there will be an example of discontinuity of landing points of periodic rays. Finally, we will outline the global topological properties of the Multicorns that we want to address later on.

Omitted values and Herman rings

Taranka Nayak (Indian Institute of Technology Bhubaneswar)

The relation between the Herman rings and omitted values of meromorphic functions is explored. In particular we have proved non existence of Herman rings for functions having at least one omitted value in some situations. If all the poles of such a function are multiple then it has

no Herman ring. Herman ring of period one or two does not exist. Functions with a single pole or with at least two poles one of which is an omitted value have no Herman ring.

Riemann surface laminations and holomorphic dynamics

Carlos Pérez (Universidad Complutense de Madrid)

We will motivate the study of laminations and foliations by Riemann surfaces, trying to explain several relationships with discrete holomorphic dynamics. Finally, we will present some results about the ergodicity of laminations embedded on certain complex surfaces.

A combinatorial point of view of some dynamic problems

Jérôme Tomasini (Université d'Angers)

The main goal of this talk is to highlight, via two exemples, the interest of choosing a combinatorial point of view to deal with dynamic problems. The two exemples that we will consider are about polynomial vector fields, and invariant laminations.

On the measure of the escaping set of a quasiregular analogue of the sine function

Sebastien Vogel (Kiel University)

Generalizing a result of McMullen (1987), we show that the escaping set of a higher dimensional quasiregular analogue of the sine function introduced by Bergweiler and Eremenko (2011) has positive Lebesgue measure.

LIST OF PARTICIPANTS

Simon Albrecht	Kiel University
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