The escaping set and spiders' webs for transcendental entire functions

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This series of lectures is concerned with the iteration of transcendental entire functions – in particular, with the *escaping set* of such a function f defined by

$$I(f) = \{z : f^n(z) \to \infty \text{ as } n \to \infty\}.$$

The first general results on this set were proved by Eremenko in 1989 and showed that the escaping set plays a fundamental role in the theory – in particular, Eremenko showed that the Julia set is always equal to the boundary of the escaping set and that the components of the closure of the escaping set are always unbounded. This led him to remark that it is plausible that the components of the escaping set are always unbounded. This is now known as Eremenko's conjecture and has motivated a large amount of work in recent years which has transformed our understanding of the structure of the escaping set.

Much work has focussed on exponential functions and other classes of functions for which the escaping set consists of a *Cantor bouquet* of curves. In joint work with Phil Rippon we have shown that there are large classes of functions for which the escaping set has a rather different structure which we call a *spider's web* – that is, the escaping set $I(f)$ is a connected set and there is a sequence of bounded simply connected domains G_n with $G_n \subset G_{n+1}$ and $\bigcup_{n\in\mathbb{N}} G_n = \mathbb{C}$ such that $\partial G_n \subset I(f)$ for all $n \in \mathbb{N}$. Eremenko's conjecture is clearly satisfied in a very strong way for such functions.

Most of our examples have the stronger property that a level of the *fast escaping set* has the structure of a spider's web and we are able to show that such functions have many strong dynamical properties – in particular, they also satisfy an apparently unrelated conjecture due to Baker concerning the components of the Fatou set.