Abstracts of short talks

Fall School on Complex Dynamics

Real analyticity of Jacobian of invariant measures for hyperbolic meromorphic functions

Badenska Agnieszka (Warsaw University of Technology)

I prove that for a hyperbolic meromorphic function f having a rapid derivative growth, if $HD(J(f)) > 1$, then the Jacobian D_{μ_ϕ} of a probability invariant measure μ_ϕ on $J(f)$, equivalent to a conformal measure m_ϕ, has a real analytic extension on a neighbourhood of $J(f) \setminus f^{-1}(\infty)$ in $\mathbb{C}$. If, in addition, f satisfies balanced derivative growth condition with constant exponents, then this extension is bounded in a neighbourhood of every pole of f.

An entire transcendental family with a fixed Siegel disk

Rubén Berenguel (Universitat de Barcelona)

This is an ongoing work with Núria Fagella devoted to the study of the entire transcendental family

$$f_{\lambda,a}(z) = \lambda a(e^{z/a}(z + 1 - a) - 1 + a), \quad \lambda = e^{2\pi i\xi}, \ \xi \text{ diophantine},$$

in particular we want to study the interplay between the critical point -1 and the asymptotic value $\lambda a(a - 1)$ with the boundary of the fixed Siegel disk around 0.

In this talk I will outline the first part of this study, which is dedicated to some properties of dynamic and parameter planes of this family with a fixed λ.

Conformal mappings and Hausdorff dimension

Albert Clop (Universitat Autnoma de Barcelona)

Conformal mappings preserve Hausdorff dimension and Hausdorff measures. In general, quasiconformal mappings may distort these set functions, although this distortion is only by a bounded amount.

We study quasiconformal mappings whose Beltrami coefficient has first order derivatives in L^2. These mappings do not change Hausdorff dimension. Further, they preserve the sets of zero length, and map the unit segment onto a rectifiable curve.

Our proofs use questions on removable singularities for analytic functions.

Local connectivity of Julia sets for some infinitely renormalizable quadratic polynomials

Dzmitry Dudko (Jacobs University at Bremen)

In 1990 Yoccoz proved the MLC Conjecture ("the Mandelbrot set is locally connected") for all parameter values which are at most finitely renormalizable. This reduced the MLC Conjecture to the infinitely renormalizable case. We will briefly discuss several results in this direction which are due to Lyubich. In particular, if the infinitely renormalizable quadratic polynomial $f = z^2 + c$ satisfies the secondary limbs condition with sufficiently big combinatorial type then the Julia set $J(f)$ will be locally connected, and the Mandelbrot set is locally connected at c. The same result will be true if the polynomial f satisfies the decoration condition.

A family of functions with a Baker domain

Dominique Fleischmann (The Open University)

Baker proved that the function f defined by

$$f(z) = z + \frac{\sin\sqrt{z}}{\sqrt{z}} + c$$

has a Baker domain for c sufficiently large. In this talk we describe the use of a novel method to prove that f has a Baker domain for all $c > 0$. The new method involves showing that there exist two unbounded simply connected open sets R and R' such that $R \subset R'$ and $f^n(z) \in R'$ whenever $z \in R$. Note that we do not require that R is invariant.

Attracting basins of trascendental functions converging to Baker Domains

Tania Garfias-Macedo (Goettingen University)

Let be a family of functions with two parameters defined as $g_{\lambda,m}(z) = (1-1/m)(z-1+\lambda ze^z)$ with $\lambda \in C \setminus \{0\}$ and $m \in N$, $m \to \infty$. We have that for every λ and large enough m, the functions have an attracting fixed point. For the limit as $m \to \infty$, the attracting basin disappears and a Baker domain takes its place. We describe briefly the dynamics of the function $g_{\lambda,m}$ in general, giving some characteristics of Fatou and Julia sets.

Julia Sets Converging to the Unit Disk

Antonio Garijo–Real (Universitat Rovira i Virgili)

We consider the family of rational maps $F_{\lambda,n,d}(z) = z^n + \lambda/z^d$ where $n, d \geq 2$ and λ is small. If λ is equal to 0 the limiting map is $F_0(z) = z^n$ and the Julia set is the unit circle. We investigate the behavior of the Julia sets of $F_{\lambda,n,d}$ when λ tends 0, obtaining two very different cases depending on n and d. The first case occurs when $n = d = 2$; here the Julia sets of $F_{\lambda,n,d}$ converge as sets to the closed unit disk. In the second case, when one of n or d is larger than 2, there is always an annulus of some fixed size in the complement of the Julia set, no matter how small $|\lambda|$ is.

Thermodynamics formalism for entire maps

Irene Inoquio (Universidad Católica del Norte)

We consider invariant subsets in code space with a countably infinite alphabet, these subsets encode the dynamics of some transcendental maps. The goal is to study the thermodynamic formalism for this class of subsets, in particular we study the symbolic dynamic of the family of maps $E_\lambda(z) := \lambda \exp(z)$, with $z \in \mathbb{C}$ and $\lambda \in (0, 1/e]$ and thus we carry the definition of topological pressure to this context and show that it satisfies a variational principle for the metric entropies, moreover we also obtain spectral consequences with the Ionescu-Tulcea and Marinescu theorem, also construct conformal measures and equilibrium measures.

Wandering Domains for Torus Diffeomorphisms

Ferry Kwakkel (Warwick University)

In dimension one, it is a well known fact that a circle homeomorphism $f : \mathbb{T}^1 \to \mathbb{T}^1$ without periodic points is semi-conjugate, through a continuous and onto map $h : \mathbb{T}^1 \to \mathbb{T}^1$ to an irrational rotation. However, this semi-conjugacy in general is not a homeomorphism, as f may have a wandering interval I, i.e. an interval $I \subset \mathbb{T}^1$ with the property that $f^n(I) \cap f^m(I) = \emptyset$ for all $n \neq m$. Clearly, the existence of a wandering interval prohibits h from being a homeomorphism.

In dimension two, homeomorphisms $f : \mathbb{T}^2 \to \mathbb{T}^2$ semiconjugate to an ergodic translation can also have a wandering domain, that is, a connected open set U, with the property that $f^n(U) \cap f^m(U) = \emptyset$ for all $n \neq m$. I will discuss the possible types of wandering domains and give a topological classification of the possible dynamics for this class of torus homeomorphisms. I will give examples of torus diffeomorphisms that show that wandering domains can also exist if f has

certain smoothness. I finish with a conjecture that says that given sufficient smoothness, f can not have a wandering domain. This conjecture, if true, would be a the two-dimensional analogue of Denjoy's Theorem in dimension one.

A Combinatorial Classification Of Postcritically Fixed Newton Maps

Yauhen Mikulich (Jacobs University at Bremen)

Aside from being a useful tool for numerical root finding, Newton maps of polynomials form an interesting subset of the space of rational functions. A number of people have used combinatorial models to structure the parameter space of Newton maps.

Tan Lei [1] gave a classification of postcritically finite cubic Newton maps in terms of matings and captures.J.Luo [2] extended some of these result to the case of Newton polynomials with one inflection value, in some sense "unicritical" Newton maps.

We present the results of J.Ruckert [3] in this direction: the theorem that structures the dynamical plane of postcritically finite Newton maps, and then use this result to construct a graph that classifies those Newton maps, whose critical orbits all terminate at fixed points.

We call a Newton map *postritically fixed* if all of its critical points are mapped onto fixed points after finitely many iterations.

Definition 1. (Newton Map) .

A rational function $f : \hat{\mathbb{C}} \to \hat{\mathbb{C}}$ of degree $d \geq 3$ is called a Newton map if ∞ is a repelling fixed point of f and for each fixed point $\xi \in \mathbb{C}$, there exists and integer $m \geq 1$ such that $f'(\xi) = (m-1)/m$.

Theorem 1. (Newton Maps Generate Newton Graphs) .

Every postcritically fixed Newton map gives rise to a unique abstract Newton graph.

If f_1 and f_2 are Newton maps with channel diagrams Δ_1 and Δ_2 such that $(\Delta_{1,N}, f_1)$ and $(\Delta_{2,N}, f_2)$ are equivalent as abstract Newton graphs, then f_1 and f_2 are affinely conjugate.

Theorem 2. (Newton Graphs Generates Newton Maps) .

Every abstract Newton graph is realized by a postcritically fixed Newton map, which is unique up to affine conjugacy. If f realizes two abstract Newton graphs (Γ_1, g_1) and (Γ_2, g_2), then the two abstract Newton graphs are equivalent.

The construction of an abstract Newton graph can be done for all postcritically finite polynomials, but will in general not contain the orbits of all critical points, and thus not describe the combinatorics of all Fatou components.

Our goal is to classify *all* Newton maps or some subclass thereof.

References.

[1] Tan Lei: *Branched coverings and cubic Newton maps.* Fund.Math (154) 1997,
207-260.

[2] J.Luo: *Newton's method for polynomials with one inflection value*, preprint, Cornell University 1993.

[3] J.Rueckert: *A Combinatorial Classification of Postcritically Fixed Newton Maps.* ArXiv: math.DS/0701176 .

Hausdorff measure of Julia sets of exponential functions

Jörn Peter (Kiel University)

We consider the exponential family, consisting of all functions

$$f_\lambda := \lambda \exp, \lambda \in \mathbb{C} \setminus \{0\}.$$

Let $\mathcal{J}(f_\lambda)$ denote the Julia set of the function f_λ. In his celebrated paper [1], McMullen shows that

(1) the Hausdorff dimension of $\mathcal{J}(f_\lambda)$ is equal to 2 for any λ
(2) the two-dimensional Hausdorff measure of $\mathcal{J}(f_\lambda)$ is 0 for any *hyperbolic* parameter λ

One can ask about the Hausdorff measure of Julia sets of exponential maps with respect to certain 'gauge functions', i.e. non-decreasing continuous functions

$$h : [0, a) \to \mathbb{R}_{\geq 0} (\text{where } a > 0 \text{ is arbitrary}) \text{ s.t. } h(0) = 0.$$

The *Hausdorff measure of* $\mathcal{J}(f_\lambda)$ *with respect to* h is then defined by

$$\mathcal{H}^h(\mathcal{J}(f_\lambda)) := \lim_{\delta\to 0} \inf \left\{ \sum_{i=1}^{\infty} h(\text{diam } A_i) : \bigcup_{i=1}^{\infty} A_i \supset \mathcal{J}(f_\lambda), \text{diam } A_i < \delta \right\}.$$

By definition, Hausdorff measure with respect to power functions $t \mapsto t^s$ is the same as s-dimensional Hausdorff measure. In [1], McMullen remarks without proof that if

$$h(t) = t^2 \cdot \log^k \left(\frac{1}{t}\right),$$

then

$$\mathcal{H}^h(\mathcal{J}(f_\lambda)) = \infty \text{ for any parameter } \lambda \in \mathbb{C} \setminus \{0\}.$$

We prove a slightly stronger statement:

Let $0 < \mu < \frac{1}{e}$, β_μ the unique real repelling fixed point of f_μ and $\gamma_\mu \in \mathbb{R}$ s.t. $\beta_\mu^{\gamma_\mu} > 2$.
Let $S_\mu : \mathbb{C} \to \mathbb{C}$ be the entire function s.t.

$$f_\mu(S_\mu(z)) = S_\mu(\beta_\mu z) \text{ for all } z \in \mathbb{C},\ S_\mu(0) = \beta_\mu \text{ and } S'_\mu(0) = 1.$$

Let

$$\Phi_\mu := (S_\mu|_{\mathbb{R}})^{-1}.$$

Then it can be shown that Φ_μ grows slower than any iterate of the logarithm. Define

$$h_\mu(t) := t^2 \cdot \Phi_\mu \left(\frac{1}{t}\right)^{\gamma_\mu}.$$

We show that $\mathcal{H}^h_\mu(\mathcal{J}(f_\lambda)) = \infty$ for any parameter $\lambda \in \mathbb{C} \setminus \{0\}$, generalizing McMullen's result.

[1] C.McMullen. Area and Hausdorff dimension of Julia sets of entire functions. Trans. Amer. Math. Soc. 300(1987), 329–342

Quasi- Fuchsian correspondences

Manjula Samarasinghe (University of London)

We consider the iterative behaviour of holomorphic correspondences or algebraic functions acting on the Riemann sphere $\overline{\mathbb{C}}$ and their limit sets. A holomorphic correspondence is a polynomial relation $P(z, w) = 0$ from $\overline{\mathbb{C}} \times \overline{\mathbb{C}}$ to $\overline{\mathbb{C}}$. We say that P is an $(n : m)$ holomorphic correspondence if the degrees of z and w are n and m respectively; the limit set of a holomorphic correspondence in this scheme is taken to be the smallest completely invariant closed set with cardinality at least three.

We identify a class of $(2:2)$ holomorphic correspondences $P(z,w) = 0$ whose limit set is a quasicircle for which the dynamical system $z \mapsto w$ is a perturbation of the action of the Modular group $\mathbf{PSL}(2,\mathbb{C})$ on $\overline{\mathbb{C}}$.

Further, we generalise these results to a class of $(3:3)$ holomorphic correspondences with analogous properties.

Baker Domains

Phil Rippon (The Open University)

This talk will provide a short survey of Baker domains and their properties, such as the growth of iterates in Baker domains, the relationship with singular values and the classification of Baker domains, illustrated by several examples.

Connectivity of Julia sets of transcendental meromorphic maps and weakly repelling fixed points

Jordi Taixés (Universitat de Barcelona)

It is known that the Julia set of the Newton's method for a non-constant polynomial is connected (M. Shishikura, 1990). This is, in fact, a consequence of a much more general result that establishes the relationship between simple connectivity of Fatou components of rational maps and fixed points which are repelling or parabolic with multiplier 1. In this talk I will speak of Fatou components of transcendental moromorphic maps instead, and show results and work in progress towards the transcendental versions of Shishikura's theorems.

TBA

Christian Henriksen (Technical University of Denmark)

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