On my eight articles with Professor Jorge Sotomayor

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Memorial Day in honor of Jorge Sotomayor
March 25, 2022

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This book helped strongly to our group in Barcelona to introduce us and work on the polynomial differential equations, one of the main areas of research of our group.

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These last years my contacts with Sotomayor were mainly during the meetings of the Oficina de Sistemas Dinamicos.

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So we have a big mathematical debt and a big friendship debt with Soto, as all his friends called him.

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A. Gasull, J. Llibre and J. Sotomayor, Further considerations on the number of vector fields of the form $X(v) = Av + f(v)Bv$, J. Differential Equations **68** (1987), 36–40.

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In the first paper we characterize under convenient assumptions the limit cycles of those differential systems when at the origin the system has a focus, and in the second one when at the origin the system has a node with different eigenvalues.

These two papers woke me up the interest for studying the limit cycles of the differential systems, one of my favourite topics of research.

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In this paper we consider an autonomous system of differential equations

$$\dot{x} = f(x), \qquad (\dot{=} d/dt), \tag{1}$$

where $x = (x_1, x_2)$ and $f(x) = (f_1(x_l, x_2), f_2(x_1, x_2))$.

Let $\mathcal{F}$ be the class of C^1 maps $f : \mathbb{R}^2 \to \mathbb{R}^2$ such that

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Fundamental Problem on Global Asymptotic Stability. Does $f \in \mathcal{F}$ imply that $x = O$ is a global asymptotically stable solution of the differential system (1)?

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Fundamental Problem on Global Asymptotic Stability. Does $f \in \mathcal{F}$ imply that $x = O$ is a global asymptotically stable solution of the differential system (1)? In other words, does every orbit of system (1) approach O as $t \to \infty$?

Problem 1. Does $f \in \mathcal{F}$ imply that the mapping $f : \mathbb{R}^2 \to \mathbb{R}^2$ is globally one-to-one?

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Problem 4. Does $f \in \mathcal{F}$ imply that

$$\int_0^\infty [\min_{|x|=r} |f(x)|] dr = \infty ?$$

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THEOREM. The following five statements are equivalent.

(FP) The Fundamental Problem has an affirmative answer for all $f \in \mathcal{F}$.

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C. Gutierrez, A solution to the bidimensional global asymptotic stability conjecture, Ann. Inst. Henri Poincaré **12** (1995), 627–671.

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In this paper we characterize the limit cycles of the planar non-smooth differential systems

$$\dot{x} = Ax + \varphi(k \cdot x)b,$$

where A is a 2×2 real matrix, $x, k, b \in \mathbb{R}^2$ and

$$\varphi(v) = \begin{cases} -u & \text{for } v \leq -u, \\ v & \text{for } -u \leq v \leq u, \\ u & \text{for } v \geq u, \end{cases}$$

where $u > 0$.

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This paper in 1996 was my first paper on non-smooth differential systems.

Just after doing this paper on non-smooth differential systems I wrote with Professor Enrique Ponce the next paper also on non-smooth differential systems:

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This paper motivates to the school of Sevilla to work on non-smooth differential systems and to produce the big number of excellent papers on those systems that they have done until now. So the influence of Soto was great and strong.

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In this paper we improve the classical Darboux theory of integrability of the polynomial differential system, showing how to use a class of equilibrium points of those systems for simplifying their integrability.

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In this paper we characterize the structural stability of the constrained polynomial differential systems of the form

$$\begin{aligned} a(x,y)\dot{x} + b(x,y)\dot{y} &= f(x,y), \\ c(x,y)\dot{x} + d(x,y)\dot{y} &= g(x,y), \end{aligned}$$

under small perturbations of the coefficients of the polynomial functions a, b, c, d, f and g.

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under small perturbations of the coefficients of the polynomial functions a, b, c, d, f and g. These systems differ from the ordinary differential equations at the impasse points defined by $ad - bc = 0$.

We extended to these constrained polynomial differential systems the results on structural stability of the smooth constrained differential systems and of the ordinary polynomial differential systems.

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J. Llibre, J. Sotomayor and M. Zhitomirskii, Impasse bifurcations of constrained systems, in Differential Equations and Dynamical Systems, Fields Institute Communications, Amer. Mat. Soc. **31** (2002), 235–255.

In that paper we studied the generic bifurcations of the families of constrained C^r differential systems of the form

$$\begin{aligned} a(x,y,\lambda)\dot{x} + b(x,y,\lambda)\dot{y} &= f(x,y,\lambda), \\ c(x,y,\lambda)\dot{x} + d(x,y,\lambda)\dot{y} &= g(x,y,\lambda), \end{aligned}$$

where $(x,y) \in \mathbb{R}^2$ and λ is a real parameter.

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where $(x,y) \in \mathbb{R}^2$ and λ is a real parameter.

The analysis of the bifurcations is located around the impase surface $ad - bc = 0$ where the constrained systems differ from the ordinary differential systems.

Finally, my last paper with Soto was also written with Professor Ronaldo Garcia:

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R. Garcia, J. Llibre and J. Sotomayor, Lines of principal curvature on canal surfaces in $\mathbb{R}^3$, An. Acad. Brasil. Ciénc. 78 (2006), no. 3, 405–415.

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R. Garcia, J. Llibre and J. Sotomayor, Lines of principal curvature on canal surfaces in $\mathbb{R}^3$, An. Acad. Brasil. Ciénc. 78 (2006), no. 3, 405–415.

In this paper we determined the principal curvatures and principal curvature lines on canal surfaces which are the envelopes of families of spheres with variable radius and centers moving along a closed regular curve in $\mathbb{R}^3$.

By means of a connection of the differential equations for these curvature lines and real Riccati differential equations, it is established that canal surfaces have at most two isolated periodic principal lines.

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Examples of canal surfaces with two simple and one double periodic principal lines were given.

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Of course the lines of curvatures was one of the main topics in the research of Soto with the collaboration of Ronando Garcia and Carlos Gutierrez.

THANK YOU VERY MUCH FOR YOUR ATTENTIONS.