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New lower bounds of the number of critical periods in reversible centers

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Abstract

In this paper we aim to find the highest number of critical periods in a class of planar systems of polynomial differential equations for fixed degree having a center. We fix our attention to lower bounds of local criticality for low degree planar polynomial centers. The main technique is the study of perturbations of reversible holomorphic (isochronous) centers, inside the reversible centers class. More concretely, we study the Taylor developments of the period constants with respect to the perturbation parameters. First, we see that there are systems of degree $3 \leq n \leq 16$ for which up to first order at least $(n^2+n-4)/2$ critical periods bifurcate from the center. Second, we improve this number for centers with degree from 3 to 9. In particular, we obtain 6 and 10 critical periods for cubic and quartic degree systems, respectively.

MSC: 34C23; 34C07

Keywords: Bifurcation of critical periods; Criticality; Period constants; Period function; Time-reversible centers

1. Introduction

Huygens, with his work on the cycloidal pendulum in the 17th century, was the forerunner of isochronicity studies and aroused the interest of this line of research, see [2]. In the last 30 years many authors have studied the existence of differential equations with equilibrium points

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