Simultaneous Bifurcation of Limit Cycles and Critical Periods

Regilene D. S. Oliveira[1] · Iván Sánchez-Sánchez[2] · Joan Torregrosa[2,3]

Abstract

The present work introduces the problem of simultaneous bifurcation of limit cycles and critical periods for a system of polynomial differential equations in the plane. The simultaneity concept is defined, as well as the idea of bi-weakness in the return map and the period function. Together with the classical methods, we present an approach which uses the Lie bracket to address the simultaneity in some cases. This approach is used to find the bi-weakness of cubic and quartic Liénard systems, the general quadratic family, and the linear plus cubic homogeneous family. We finish with an illustrative example by solving the problem of simultaneous bifurcation of limit cycles and critical periods for the cubic Liénard family.

1 Introduction

A classical problem in the study of qualitative theory of planar differential equations in the plane is the 16th Hilbert Problem, related to the bifurcation of limit cycles (isolated

✉ Joan Torregrosa
   torre@mat.uab.cat

   Regilene D. S. Oliveira
   regilene@icmc.usp.br

   Iván Sánchez-Sánchez
   isanchez@mat.uab.cat

[1] Departamento de Matemática, Instituto de Ciências Matemáticas e Computação, Universidade de São Paulo, Avenida Trabalhador São Carlense, São Carlos, SP, 400 13566-590, Brazil

[2] Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain

[3] Centre de Recerca Matemàtica, Campus de Bellaterra, 08193 Bellaterra, Barcelona, Spain

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