Separatrix surfaces and invariant manifolds of a class of integrable Hamiltonian systems and their perturbations

Memòria presentada per Ana Maria Ribeiro Ferreira Nunes per a aspirar al grau de doctor en Ciències Matemàtiques.

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Bellaterra, Setembre de 1989.

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