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The criticality of reversible quadratic centers at the outer boundary of its period annulus ☆

D. Marín [a,b], J. Villadelprat [c,*]

[a] Departament de Matemàtiques, Edifici Cc, Universitat Autònoma de Barcelona, 08193 Cerdanyola del Vallès, Barcelona, Spain

[b] Centre de Recerca Matemàtica, Edifici Cc, Campus de Bellaterra, 08193 Cerdanyola del Vallès, Barcelona, Spain

[c] Departament d'Enginyeria Informàtica i Matemàtiques, ETSE, Universitat Rovira i Virgili, 43007 Tarragona, Spain

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Abstract

This paper deals with the period function of the reversible quadratic centers

$$X_\nu = -y(1-x)\partial_x + (x + Dx^2 + Fy^2)\partial_y,$$

where $\nu = (D, F) \in \mathbb{R}^2$. Compactifying the vector field to $\mathbb{S}^2$, the boundary of the period annulus has two connected components, the center itself and a polycycle. We call them the inner and outer boundary of the period annulus, respectively. We are interested in the bifurcation of critical periodic orbits from the polycycle Π_ν at the outer boundary. A critical period is an isolated critical point of the period function. The criticality of the period function at the outer boundary is the maximal number of critical periodic orbits of X_ν that tend to Π_{ν_0} in the Hausdorff sense as $\nu \to \nu_0$. This notion is akin to the cyclicity in Hilbert's 16th Problem. Our main result (Theorem A) shows that the criticality at the outer boundary is at most 2 for all $\nu = (D, F) \in \mathbb{R}^2$ outside the segments $\{-1\} \times [0, 1]$ and $\{0\} \times [0, 2]$. With regard to the bifurcation from the inner boundary, Chicone and Jacobs proved in their seminal paper on the issue that the upper bound is 2 for all $\nu \in \mathbb{R}^2$. In this paper the techniques are different because, while the period function extends analytically to the center, it has no smooth extension to the polycycle. We show that the period function has an asymptotic expansion near the polycycle with the remainder being uniformly flat with respect to ν

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* Corresponding author.

E-mail addresses: davidmp@mat.uab.cat (D. Marín), jordi.villadelprat@urv.cat (J. Villadelprat).

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