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On the cyclicity of Kolmogorov polycycles

David Marín[1,2] and **Jordi Villadelprat**[✉ 3]

[1]Departament de Matemàtiques, Edifici Cc, Universitat Autònoma de Barcelona, 08193 Cerdanyola del Vallès (Barcelona), Spain
[2]Centre de Recerca Matemàtica, Edifici Cc, Campus de Bellaterra, 08193 Cerdanyola del Vallès (Barcelona), Spain
[3]Departament d'Enginyeria Informàtica i Matemàtiques, ETSE, Universitat Rovira i Virgili, 43007 Tarragona, Spain

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Abstract. In this paper we study planar polynomial Kolmogorov's differential systems

$$X_\mu \quad \begin{cases} \dot{x} = xf(x,y;\mu), \\ \dot{y} = yg(x,y;\mu), \end{cases}$$

with the parameter μ varying in an open subset $\Lambda \subset \mathbb{R}^N$. Compactifying X_μ to the Poincaré disc, the boundary of the first quadrant is an invariant triangle Γ, that we assume to be a hyperbolic polycycle with exactly three saddle points at its vertices for all $\mu \in \Lambda$. We are interested in the cyclicity of Γ inside the family $\{X_\mu\}_{\mu\in\Lambda}$, i.e., the number of limit cycles that bifurcate from Γ as we perturb μ. In our main result we define three functions that play the same role for the cyclicity of the polycycle as the first three Lyapunov quantities for the cyclicity of a focus. As an application we study two cubic Kolmogorov families, with $N = 3$ and $N = 5$, and in both cases we are able to determine the cyclicity of the polycycle for all $\mu \in \Lambda$, including those parameters for which the return map along Γ is the identity.

Keywords: limit cycle, polycycle, cyclicity, asymptotic expansion.

2020 Mathematics Subject Classification: 34C07, 34C20, 34C23.

1 Introduction and main results

The present paper is motivated by the results obtained by Gasull, Mañosa and Mañosas [8] with regard to the *stability* of an unbounded polycycle Γ in the Kolmogorov's polynomial differential systems

$$\begin{cases} \dot{x} = xf(x,y), \\ \dot{y} = yg(x,y). \end{cases}$$

These systems are widely used in ecology to describe the interaction between two populations, see [18] for instance. That being said, the stability of the polycycle is not the main issue to

✉ Corresponding author. Email: jordi.villadelprat@urv.cat