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Asymptotic expansion of the Dulac map and time for unfoldings of hyperbolic saddles: General setting ☆

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Abstract

Given a $\mathscr{C}^\infty$ family of planar vector fields $\{X_{\hat{\mu}}\}_{\hat{\mu}\in\hat{W}}$ having a hyperbolic saddle, we study the Dulac map $D(s;\hat{\mu})$ and the Dulac time $T(s;\hat{\mu})$ between two transverse sections located in the separatrices at arbitrary distance from the saddle. We show (Theorems A and B, respectively) that, for any $\hat{\mu}_0\in\hat{W}$ and $L>0$, the functions $T(s;\hat{\mu})$ and $D(s;\hat{\mu})$ have an asymptotic expansion at $s=0$ for $\hat{\mu}\approx\hat{\mu}_0$ with the remainder being uniformly L-flat with respect to the parameters. The principal part of both asymptotic expansions is given in a monomial scale containing a deformation of the logarithm, the so-called Roussarie-Ecalle compensator. The coefficients of these monomials are $\mathscr{C}^\infty$ functions "universally" defined, meaning that their existence is established before fixing the flatness L of the remainder and the unfolded parameter $\hat{\mu}_0$. Moreover the flatness L of the remainder is preserved after any derivation with respect to the parameters. We also provide (Theorem C) an explicit upper bound for the number of zeros of $T'(s;\hat{\mu})$ bifurcating from $s=0$ as $\hat{\mu}\approx\hat{\mu}_0$. This result enables to tackle finiteness problems for the number of critical periodic orbits along the lines of those theorems on finite cyclicity around Hilbert's 16th problem. As an application we prove two finiteness results (Corollaries D and E) about the number of critical periodic orbits of polynomial vector fields.

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