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Journal of Differential Equations 380 (2024) 181-197

Journal of Differential Equations

www.elsevier.com/locate/jde

Critical periods in planar polynomial centers near a maximum number of cusps

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> Received 21 June 2023; accepted 18 October 2023 Available online 7 November 2023

Abstract

We provide the best lower bound for the number of critical periods of planar polynomial centers known up to now. The new lower bound is obtained in the Hamiltonian class and considering a single period annulus. This lower bound doubles the previous one from the literature, and we end up with at least $n^2 - 2$ (resp. $n^2 - 2n - 1$) critical periods for planar polynomial systems of odd (resp. even) degree n. Key idea is the perturbation of a vector field with many cusp equilibria, whose construction is by itself a nontrivial construction that uses elements of catastrophe theory. © 2023 Elsevier Inc. All rights reserved.

MSC: primary 34C07; secondary 34C23, 37C27

Keywords: Critical periods; Hamiltonian vector fields; Best lower bound; Degree n vector field

1. Introduction

The study of the number of oscillations of the period function associated to a planar polynomial center has a long history that took a start with the seminal paper by Chicone and Jacobs in 1989 where the authors did a bifurcation analysis of the critical periods for planar vector fields in [2]. In classical Liénard families the isochronicity and monotonicity problems were studied

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https://doi.org/10.1016/j.jde.2023.10.034

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