



Darboux theory of integrability in \mathbb{T}^n

Jaume Llibre¹ · Claudia Valls²

Received: 12 December 2022 / Revised: 30 January 2024 / Accepted: 25 February 2024
© The Author(s) 2024

Abstract

We develop the Darboux theory of integrability for polynomial vector fields in the n -dimensional torus \mathbb{T}^n . We also provide the maximum number of invariant parallels and meridians that a polynomial vector field X on \mathbb{T}^n can have in function of its degree.

Keywords Darboux integrability · Tori · Invariant algebraic variety · Exponential factor

Mathematics Subject Classification 34C05

1 Introduction and statement of main results

Real nonlinear ordinary differential systems are used to model a wide range of processes practically in all fields of science, from biology and chemistry to economy, physics and engineering. The existence of first integrals of differential systems defined on \mathbb{R}^n is important for two main reasons. First, they make easier the characterization of the phase portrait of the system. Secondly, their existence allows reducing the dimension of the system by its number of independent first integrals, which in many cases makes easier the analysis. In our terminology, a system is integrable if it has $n - 1$ independent first integrals if the space has dimension n . Therefore the meth-

Jaume Llibre is supported by the Agencia Estatal de Investigación Grant PID2019-104658GB-I00 and the H2020 European Research Council Grant MSCA-RISE-2017-777911. Claudia Valls is partially supported by FCT/Portugal through CAMGSD, IST-ID, Projects UIDB/04459/2020 and UIDP/04459/2020.

✉ Jaume Llibre
jllibre@mat.uab.cat

Claudia Valls
cvalls@math.ist.utl.pt

¹ Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain

² Departamento de Matemática, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais 1049-001, Lisbon, Portugal