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The improved Euler–Jacobi formula and the planar cubic polynomial vector fields in $\mathbb{R}^2$

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ABSTRACT

The new Euler–Jacobi formula for points with multiplicity two provides an algebraic relation between the singular points of a polynomial vector field and their topological indices. Using this formula we obtain the configuration of the singular points together with their topological indices for the planar cubic polynomial differential systems when these systems have eight finite singular points, being one of them with multiplicity two. The case with nine finite singular points has already been solved using the classical Euler–Jacobi formula.

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1. Introduction and statement of the main results

Consider the planar polynomial differential system

$$\dot{x} = P(x,y), \quad \dot{y} = Q(x,y), \tag{1}$$

in $\mathbb{R}^2$ where $P(x,y)$ and $Q(x,y)$ are real polynomials of degree n and m, respectively. The vector field associated to the planar polynomial differential system (1) will be denoted by X.

When $n = m = 2$ and system (1) has four singular points the possible configurations of the (topological) indices of these points where characterized by Berlinskii in [2] in 1960. Later on in 1999 Gasull and Torregrosa [5] also for systems (1) with $n = m = 2$ but with three singular points one with multiplicity two extended the Berlinskii theorem to these systems. More recently, the authors in [8] with $n = m = 3$ (i.e. for cubic polynomial differential systems) classify all the possible configurations of the indices of these systems having nine singular points, see also [7,9]. The objective of this paper is to classify all the configurations of indices of systems (1) with $n = m = 3$ having eight singular points one of them with multiplicity two.

When system (1) has nm finite singular points the classical Euler–Jacobi formula (see [1] for its proof) gives an algebraic relation between the indices of these finite singular points. When system (1) has points with multiplicity two, the classical Euler–Jacobi formula is no longer valid. However in [5] the authors (Gasull and Torregrosa) provided a generalization of the classical Euler–Jacobi formula when the system has points with multiplicity two. Using this new formula and the index theory we obtain the distribution of the

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