



Non-existence and uniqueness of limit cycles in a class of generalized Liénard equations

Jaume Llibre¹ · Claudia Valls²

Received: 7 October 2021 / Accepted: 12 April 2022 / Published online: 12 May 2022
© Sociedad Matemática Mexicana 2022

Abstract

We provide a sharp upper bound for the number of limit cycles of the generalized Liénard differential systems

$$\dot{x} = y + ax^n + bx^k, \quad \dot{y} = cx^m$$

where n, k, m are positive integers, $1 < n < k$ and $a, b, c \in \mathbb{R}$ with $bc \neq 0$. We also provide examples realizing the upper bounds

Keywords Liénard equations · Limit cycles · Periodic orbits

Mathematics Subject Classification 34C05

1 Introduction and statement of the main results

The so-called Liénard differential systems introduced by Liénard in [6] and its generalizations have been studied by many different authors, nowadays in MathSciNet appear 1285 papers related with these systems.

This paper deals with the problem of finding upper bounds for the number of limit cycles of some generalized Liénard systems. The number of limit cycles for polynomial differential systems is part of the 16th problem of the 23 problems proposed by Hilbert in 1900, see [4]. Over the last century, the uniqueness of limit

✉ Claudia Valls
cvalls@math.ist.utl.pt

Jaume Llibre
jllibre@mat.uab.cat

¹ Departament de Matemàtiques, Universitat Autònoma de Barcelona, Bellaterra, 08193 Barcelona, Catalonia, Spain

² Departamento de Matemática, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais, 1049–001 Lisboa, Portugal