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## PHASE PORTRAITS OF THE LESLIE-GOWER SYSTEM\*

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**Abstract** In this paper we characterize the phase portraits of the Leslie-Gower model for competition among species. We give the complete description of their phase portraits in the Poincaré disc (i.e., in the compactification of  $\mathbb{R}^2$  adding the circle  $\mathbb{S}^1$  of the infinity) modulo topological equivalence.

It is well-known that the equilibrium point of the Leslie-Gower model in the interior of the positive quadrant is a global attractor in this open quadrant, and in this paper we characterize where the orbits attracted by this equilibrium born.

Key words predator-prey models; Leslie–Gower system; Poincaré compactification; global phase portraits

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## 1 Introduction and Statement of the Main Results

The dynamical relations between predators and prey is one of the fundamental objects of study in population dynamics. This is mainly due to the fact that to its big number of applications [3] and because it allows a better understanding of the behavior of food chains or trophic networks (see for instance [14, 24]).

The first predator-prey model was proposed by the italian mathematician Vito Volterra [1, 2] in its celebrate publish monograph in 1926 [25] where he described the model as a nonlinear system of ordinary differential equations. This model coincided with a bidimensional model for biochemic interactions proposed earlier by the american physicist Alfred J. Lotka and this is the main reason why these type of models are called Lotka-Volterra models [2, 14, 15, 24].

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