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Limit Cycles of Planar Piecewise Differential Systems with Linear Hamiltonian Saddles

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Abstract: We provide the maximum number of limit cycles for continuous and discontinuous planar piecewise differential systems formed by linear Hamiltonian saddles and separated either by one or two parallel straight lines. We show that when these piecewise differential systems are either continuous or discontinuous and are separated by one straight line, or are continuous and are separated by two parallel straight lines, they do not have limit cycles. On the other hand, when these systems are discontinuous and separated by two parallel straight lines, we prove that the maximum number of limit cycles that they can have is one and that this maximum is reached by providing an example of such a system with one limit cycle. When the line of discontinuity of the piecewise differential system is formed by one straight line, the symmetry of the problem allows to take this straight line without loss of generality as the line $x = 0$. Similarly, when the line of discontinuity of the piecewise differential system is formed by two parallel straight lines due to the symmetry of the problem, we can assume without loss of generality that these two straight lines are $x = \pm 1$.

Keywords: crossing limit cycles; linear Hamiltonian saddles; continuous piecewise linear differential systems; discontinuous piecewise differential systems

MSC: 34C07, 34C25

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1. Introduction

The study of limit cycles of differential systems in $\mathbb{R}^2$ (i.e., periodic orbits of a differential system in $\mathbb{R}^2$ isolated in the set of all periodic orbits of that system) goes back essentially to Poincaré [1] at the end of the nineteenth century and their existence became important in application due to their relation with real world phenomena, see for instance the limit cycle of van der Pol equation [2,3], or the one of the Belousov–Zhavotinskii model [4,5].

Continuous piecewise linear differential systems separated by straight lines appear naturally in control theory (see, for instance, Refs. [6–11]). The easiest continuous piecewise linear differential systems are the ones formed by two linear differential systems separated by a straight line and for such systems it is well known that one is the upper bound on the number of limit cycles that they can have and that this upper bound is reached (see, for instance, Refs. [12–15] and the references therein).

The unique linear differential systems which are Hamiltonian are linear centers and linear saddles. In [16] the authors obtained the maximum number of limit cycles of continuous and discontinuous piecewise differential systems formed by linear centers and separated by either one or two parallel straight lines. In the present paper we do a symmetric study for continuous and discontinuous piecewise differential systems formed by linear Hamiltonian saddles and separated by either one or two parallel straight lines.