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Weierstrass Integrability of Complex Differential Equations

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Abstract We characterize the complex differential equations of the form

$$\frac{dy}{dx} = a_n(x)y^n + a_{n-1}(x)y^{n-1} + \cdots + a_1(x)y + a_0(x),$$

where $a_j(x)$ are meromorphic functions in the variable x for $j = 0, \ldots, n$ that admit either a Weierstrass first integral or a Weierstrass inverse integrating factor.

Keywords Weierstrass first integrals, Weierstrass inverse integrating factor, complex differential equations

MR(2010) Subject Classification 34C05, 34A34, 34C14

1 Introduction and Statement of the Main Results

Let x and y be complex variables. In this paper we study the differential equations of the form

$$\frac{dy}{dx} = a_n(x)y^n + a_{n-1}(x)y^{n-1} + \cdots + a_1(x)y + a_0(x) \quad \text{with } a_n(0) \not\equiv 0, \tag{1.1}$$

where $a_j(x)$ are meromorphic functions of x for $j = 0, \ldots, n$. In particular, the differential equation (1.1) contains the well-known *Abel differential equations* when $n = 3$, the *Riccati differential equations* when $n = 2$, and the *linear differential equations* when $n = 1$.

In what follows instead of working with the differential equation (1.1) we shall work with the equivalent differential system

$$\dot{x} = 1, \quad \dot{y} = a_n(x)y^n + a_{n-1}(x)y^{n-1} + \cdots + a_1(x)y + a_0(x) \quad \text{with } a_n(0) \not\equiv 0 \tag{1.2}$$

in $\mathbb{C}^2$, where the dot denotes derivative with respect to the time t, real or complex.

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